# Multi-Period Loans, One-Period Loans, Roll-over Loans, Bridge Loans

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This file discusses the role of bridge loan in improving consumption smoothing when only one period loans with roll-over is allowed in a 3 period deterministic model with fixed but unevenly distributed endowment. Development banks could be offering these one-period loans without roll-over, and informal lenders could be offering bridge loans. Informal loans with higher interest rates co-exist with formal loans in this environment.

Note that this is a context without default. Households always pay back their debts.

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# **Optimal Choices in a Three Period Model With Fixed Endowments**

Suppose there are three periods, there are fixed endowments in each period:

- $\mathbf{e} = \{e_1, e_2, e_3\}$
- $e_1 = 0$
- $e_2 = 1$
- $e_3 = 8$

What is the standard budget constraint we write down for this problem and what does it imply for the length of loan? Let's abstract away from interest rate and discount,  $\beta = 1$ , and r = 0. With log utility, we have:

$$\max_{b_2, b_3} \log(c_1) + \log(c_2) + \log(c_3)$$

such that: 
$$c_1 + b_2 = e_1$$
,  $c_2 + b_3 = e_2 + b_2$ , and  $c_3 = e_3 + b_3$ 

the implied non-binding *natural borrowing constraints* are:  $b_2 > -1 \cdot (e_2 + e_3)$  and,  $b_3 > -1 \cdot (e_3)$ 

Combining the budgets together:

$$c_1 + c_2 + c_3 = e_1 + e_2 + e_3$$

Endowments over the three periods can be shared across the three periods, optimal choices are:

$$b_2^* = -3, b_3^* = -5$$

$$c_1^* = c_2^* = c_3^* = \frac{0+1+8}{3} = 3$$

# Multi-Period Loan and One-Period Loan with Roll-over

The standard budget constraint that we write down as shown above describes short-term loans that allow for roll-overs. One-Period loan with roll-over in this case achieves the same smoothing as Multi-Period loan + savings. In both cases, we get the optimal choices as shown above.

- Multi-Period Loan: If borrowing in the 1st period is due in the 3rd period, that is a multi-period loan.
- **One-Period Loan**: Borrowing in t is due in t + 1
- Roll-over: You are allowed to repay existing debt with new debt.

## Smoothing using Multi-Period Loan and savings:

Multi-Period borrowing + savings achieves consumption smoothing (with b denoting just borrowing, and s just savings):

$$c_1 + b_2 + s_2 = e_1$$
,  $c_2 + b_3 + s_3 = e_2 + b_2 + s_2$ , and  $c_3 = e_3 + b_3 + s_3$ 

- 1. In the first period, borrow 5 units of Multi-Period loan and save 1 unit:  $b_2 = -5$ ,  $s_2 = 2$
- 2. In the second period, carry 5 units of Multi-Period loan debt to 3rd period:  $b_3 = -5$ ,  $s_3 = 0$
- 3. pay off 5 unit Multi-Period debt in the 3rd period

#### Smoothing using One-Period loan with Roll-overs:

This is implicitly what the standard budget constraint above allows for:

$$c_1 + b_2 = e_1$$
,  $c_2 + b_3 = e_2 + b_2$ , and  $c_3 = e_3 + b_3$ 

- 1. borrow 3 units of One-Period loan in t = 1:  $b_2 = -3$
- 2. borrow 5 units of One-Period loan in t = 2 to repay  $b_2 = -3$  debt and add 2 units to  $c_2$  consumption:  $b_3 = -5$
- 3. pay off 5 units of One-Period loan in t = 3

## **One-Period Loan without Roll-over**

The budget constraints when we have no-roll-over one-period loans is the same as before:

We still have: 
$$c_1 + b_2 = e_1$$
,  $c_2 + b_3 = e_2 + b_2$ , and  $c_3 = e_3 + b_3$ 

but now we have these borrowing constraints:

$$b_2 \ge -e_2$$
, and  $b_3 \ge -e_3$ 

Households still face the same natural borrowing constraint as before, but they now face the tigether constraint imposed by the bank that requires principal repayments before new loans are issued. These bank short-term repayment constraints could bind. In our example here, households only have  $e_2 = 1$ , limiting their borrowing to below this level, but they want to move 3 units over to first period for consumption.

Optimal choices now with no-roll-over one-period loan is:

$$b_2^* = -1, b_3^* = -4$$
  
 $c_1^* = 1, c_2^* = 4, c_3^* = 4$ 

Smoothing between the three periods is no longer achieved.

## **Bridge Loan**

It is common for banks to issue one year loans which require the repayment of principles and interest rates after 12 months. 12 month could be a model period, so what could households do to improve smoothing?

Suppose there is a high interest rate (informal) lender:  $r_{\rm B} = 0.75$ , 75 percent interest rate. This lender is unconcerned about lending to a household with outstanding loans. This informal lender *B* could lend a little bit to the borrower, to help the borrower bridge over and pay for the existing debt from formal bank *A*, which offers lower interest rates loans. Informal lender *B* is willing to issue new debt before borrower has paid off their existing interests + principles.

Our budgets now are:

$$c_{1} + b_{2}^{A} + b_{2}^{B} = e_{1}$$

$$c_{2} + b_{3}^{A} + b_{3}^{B} = e_{2} + b_{2}^{A} + b_{2}^{B} \cdot (1 + r_{B})$$

$$c_{3} = e_{3} + b_{3}^{A} + b_{3}^{B} \cdot (1 + r_{B})$$

And the borrowing constraints are:

debt is less than endowment and new *B* borrowing:  $b_2^A + b_2^B \cdot (1 + r_B) \ge -e_2 + b_3^B$ 

and 
$$b_3^A + b_3^B \cdot (1 + r_{\mathrm{B}}) \geq -e_3$$

# **Optimization With Bridge Loan**

Given that loan type *B* is a lot more expensive than *A*, you will only use it for bridge-loan purposes, and will not choose to borrow from source *B* in the first period. Additionally, we also know eactly how much you will borrow from source *B*, you will only borrow exactly enough to cover you unpaid principle from borrowing  $b_2^A$  given  $e_2$ . What these mean is that we can write down the following:

$$c_{1} + b_{2}^{A} = e_{1}$$

$$c_{2} + b_{3}^{A} + b_{3}^{B} = e_{2} + b_{2}^{A}$$

$$e_{2} + b_{2}^{A} - b_{3}^{B} = 0$$

$$c_{3} = e_{3} + b_{3}^{A} + b_{3}^{B} \cdot (1 + r_{B})$$

Jointly, what these imply is a new aggregate budget constraint given which we can solve the maximization problem with bridge loans:

$$\max_{b_2, b_3} \log(c_1) + \log(c_2) + \log(c_3)$$

$$c_1 \cdot (1 + r_B) + c_2 + c_3 = e_2 \cdot (1 + r_B) + e_3$$

with  $r^B = 0.75$ , we now achieve better smoothing than when bridge loans are not available:

$$b_2^{A,*} = -1.86, b_3^{B,*} = -0.86, b_3^{A,*} = -3.25$$
  
 $c_1^* = 1.86, c_2^* = 3.25, c_3^* = 3.25$ 

```
% Variables and Parmaeters
syms c1 c2 c3 r e2 e3 lambda
r_num = 0.75; % r_num = 0.75, then bridge loan demand = 0
e2_num = 1;
e3_num = 8;
% Lagrangian
lagrangian = log(c1) + log(c2) + log(c3) - lambda*(e2*(1+r) + e3 - c1*(1+r) - c2 - c3);
% Analytical Solutions
solu_max = solve(diff(lagrangian, c1)==0,...
diff(lagrangian, c2)==0,...
diff(lagrangian, c3)==0,...
diff(lagrangian, c3)==0,...
c1, c2, c3, lambda,...
'Real', true);
solu_c1 = solu_max.c1
```

solu\_c1 =

 $\frac{e_2 + e_3 + e_2 r}{3 r + 3}$ 

solu c2 = solu max.c2

solu\_c2 =

 $\frac{e_2}{3} + \frac{e_3}{3} + \frac{e_2 r}{3}$ 

solu\_c3 = solu\_max.c3

solu\_c3 =

 $\frac{e_2}{3} + \frac{e_3}{3} + \frac{e_2 r}{3}$ 

```
% Numerical Solutions
opti_c1 = double(subs(solu_c1, {r, e2, e3}, {r_num, e2_num, e3_num}));
opti_c2 = double(subs(solu_c2, {r, e2, e3}, {r_num, e2_num, e3_num}));
opti_c3 = double(subs(solu_c3, {r, e2, e3}, {r_num, e2_num, e3_num}));
% First period borrowing fully finances c1 consumption, only borrow from source A with 0 interest
opti_bA_t2 = double(-opti_c1);
% Second period borrowing first borrow from bridge loan to fill gap
opti_bB_t3 = double(opti_bA_t2 + e2_num);
% Second period consumption fully financial by borrowing from source A wiht 0 interest.
opti_bA_t3 = double(-opti_c2);
% Display results
outcomes_bridge = table(opti_c1, opti_c2, opti_c3, opti_bA_t2, opti_bB_t3, opti_bA_t3);
```

opti_c1	opti_c2	opti_c3	opti_bA_t2	opti_bB_t3	opti_bA_t3
1.8571	3.25	3.25	-1.8571	-0.85714	-3.25

#### Wealth Recovery from Bridge Loan with No-Roll-Over One Period Loans

We have these three cases:

- <sup>1</sup>. Multi-Period loan (borrow + save) or One-Period loan with roll-over:  $c_1^* = c_2^* = c_3^* = 3$
- 2. One-Period loan without roll-over:  $c_1^* = 1$ ,  $c_2^* = 4$ ,  $c_3^* = 4$
- 3. By allowing for bridge loan, we can move from the second world of imperfect smoothing towards the first world of perfect smoothing.

If  $r_B = 0$ , then we are back to perfect smoothing, given the endowment sequence here, if  $r_B = 3.00$ , which

is 300 percent interest rate, bridge loan demand is zero, and we are back to the no-roll-over One-Period loan world. With bridge loan, very high interest rate informal loans can co-exist with very low interest rate formal loans.

Solving the expenditure minimization problem, we can solve for the wealth loss due to not allowing for roll-over, and the wealth recovery achieved when bridge loans are possible. The recovery varies depending on interest rate.

#### 1. First define Parameters:

```
% Total wealth avaialble
e1 = 0;
e2 = 1;
e3 = 8;
e_total = e1 + e2 + e3;
% Loop over variations in interest rates of the bridge loan
r_min = 0; % no better than main/formal loan r
r_max = 3; % if higher than this, want to save
bridge_loan_r_vector = linspace(r_min, r_max, 100);
```

#### 2. Utility Maximization and Expenditure Minimization

```
% Symbols
syms c1 c2 c3 lambda
% Utility Function
UTIL = log(c1) + log(c2) + log(c3);
% Lagrangian for Utility Maximization with Bridge Loan
lagrangian = UTIL - lambda*(e2*(1+r) + e3 - c1*(1+r) - c2 - c3);
% Analytical Solutions
solu_max = solve(diff(lagrangian, c1)==0,...
diff(lagrangian, c2)==0,...
diff(lagrangian, c3)==0,...
diff(lagrangian, lambda)==0,...
c1, c2, c3, lambda,...
'Real', true);
solu_c1 = solu_max.c1;
```

```
solu_c2 = solu_max.c2;
solu c3 = solu max.c3;
syms util
% Lagrangian for the unconstrained original Expenditure Minimization problem
lagrangian = (c1 + c2 + c3 - lambda*(log(c1) + log(c2) + log(c3) - util));
% Expenditure Minimization Solutions
solu min = solve(diff(lagrangian, c1)==0,...
                 diff(lagrangian, c2)==0,...
                 diff(lagrangian, c3)==0,...
                 diff(lagrangian, lambda)==0,...
                 c1, c2, c3, lambda,...
                 'Real', true);
solu min c1 = solu min.c1;
solu min c2 = solu min.c2;
solu min c3 = solu min.c3;
f_total_budget = solu_min_c1 + solu_min_c2 + solu_min_c3;
```

#### 3. Loop Over Interest Rates

```
% Solution over interest rate
value_vec = zeros(size(bridge_loan_r_vector));
bridge_frac_b_vec = zeros(size(bridge_loan_r_vector));
total b vec = zeros(size(bridge loan r vector));
c1_o_vec = zeros(size(bridge_loan_r_vector));
c2_o_vec = zeros(size(bridge_loan_r_vector));
for bridge_r_ctr=1:length(bridge_loan_r_vector)
    bridge_r = bridge_loan_r_vector(bridge_r_ctr);
    % Optimal choices given interest rate
    c1_o = double(subs(solu_c1, {r}, {bridge_r}));
    c2_o = double(subs(solu_c2, {r}, {bridge_r}));
    c3_o = double(subs(solu_c3, {r}, {bridge_r}));
    % Store Util at optimal choices
    value_vec(bridge_r_ctr) = double(subs(UTIL, {c1,c2,c3}, {c1_o, c2_o, c3_o}));
    % Store Bridge Loan Choices to cover short fall as a fraction of total second period borrowing
    % Consider principle + interest
    % e2 - c1_o is bridge loan amount, because c1_o = b2.
    % And since all e2 is used to repay loans, c2 o is regular loan second period borrowing.
    bridge_frac_b_vec(bridge_r_ctr) = (abs(e2 - c1_o))/(abs(e2 - c1_o) + abs(c2_o));
    total_b_vec(bridge_r_ctr) = (e2 - c1_o) -(c2_o);
    % Optimal consuption choices for t1 and t2
    c1 \circ vec(bridge r ctr) = c1 \circ;
    c2_o_vec(bridge_r_ctr) = c2_o;
end
```

# Graphing the Effects of Bridge Loans with Varying Interest Rates

Two graphs below, first wealth loss recovery achieved by using bridge loans, and then consumption in the three periods with bridge loan given different interest rates.

Without bridge loan, 16% wealth loss (from exp. min.) due to having no-roll-over one-period loans compared to
perfect smoothing which is achieved by roll-over loans.

- If bridge loans are available, no-roll-over one-period + bridge loan moves consumption much closer to perfect smoothing achieved by roll-over loans.
- If informal bridge loan interest rate is the same as no-roll-over one period formal loan, perfect smoothing is achieved.
- Bridge loan helps to improve consumption smoothing, given the parameters here, as long as its interest rate is below 300 percent.

```
% Budget Cost to Achieve the utility vector given varying bridge loan Interests
budget vec = double(subs(f total budget, {util}, {value vec}));
% Plot Margins
r_graph_margin = (r_max - r_min)/10;
r_graph_max = r_max + r_graph_margin;
r graph_min = r_min - r_graph_margin;
x_title_line = ['(Informal) Bridge Loan Interest Rate, 1 = 100%'];
% wealth loss Plot
figure();
subplot(1,2,1);
hold on;
% wealth loss share compared to if roll-over loans are allowed
inc_loss = (budget_vec/e_total - 1)*100;
% Plot
plot(bridge_loan_r_vector, inc_loss);
% graph x and y limits
iloss_margin = (0 - min(inc_loss))/10;
iloss_graph_max = 0 + iloss_margin;
iloss_graph_min = min(inc_loss) - iloss_margin;
xlim([r graph min r graph max]);
ylim([iloss_graph_min iloss_graph_max]);
% Extra Lines
plot(0*ones(1,10), linspace(iloss_graph_min, iloss_graph_max,10), 'k-', 'LineWidth', 1);
% Labels
title({['% Wealth Loss vs. Full Smoothing'], ['Bridge + no-roll-over Formal Loans'],...
       ['e1 = ' num2str(e1)...
         , e2 = ' num2str(e2) ...
         , e3 = ' num2str(e3) ]});
xlabel(x_title_line);
ylabel({['Exp. Min. % Wealth Loss'] ['vs. Perfect Smoothing']});
% grids
grid on;
grid minor;
subplot(1,2,2);
hold on;
plot(bridge_loan_r_vector, bridge_frac_b_vec);
% graph x and y limits
brdgfrac_margin = (max(bridge_frac_b_vec) - 0)/10;
brdgfrac_graph_max = max(bridge_frac_b_vec) + brdgfrac_margin;
brdgfrac graph min = 0 - brdgfrac margin;
xlim([r graph min r graph max]);
ylim([brdgfrac_graph_min brdgfrac_graph_max]);
% Extra Lines
plot(0*ones(1,10), linspace(brdgfrac_graph_min, brdgfrac_graph_max,10), 'k-', 'LineWidth', 1);
plot(linspace(r_graph_min, r_graph_max, 10), ones(10) * 0, 'k-', 'LineWidth', 1);
% Labels
title({['Fraction of Bridge Loan in Total Loans'], ['Bridge + no-roll-over Formal Loans'],...
```

```
['e1 = ' num2str(e1)...
    ', e2 = ' num2str(e2) ...
    ', e3 = ' num2str(e3) ]});
xlabel(x_title_line);
ylabel({['Bridge Loan Share'] ['of Total Loans']});
% grids
grid on;
grid minor;
```



```
figure();
hold on;
plot(bridge_loan_r_vector, c1_o_vec);
plot(bridge_loan_r_vector, c2_o_vec);
% graph x and y limits
c12 max = max([c1 o vec c2 o vec]);
c12_min = min([c1_o_vec c2_o_vec]);
c12_margin = (c12_max - c12_min)/10;
c12_graph_max = max([c1_o_vec c2_o_vec]) + c12_margin;
c12_graph_min = min([c1_o_vec c2_o_vec]) - c12_margin;
xlim([r graph min r graph max]);
ylim([c12_graph_min c12_graph_max]);
% Extra Lines
plot(linspace(r_graph_min, r_graph_max, 10), ones(10) * 3, 'k:');
plot(linspace(r_graph_min, r_graph_max, 10), ones(10) * 4, 'k--');
plot(linspace(r_graph_min, r_graph_max, 10), ones(10) * 1, 'k--');
plot(0*ones(1,10), linspace(c12 graph min, c12 graph max,10), 'k-', 'LineWidth', 1);
% Labels
title({['Consumption with Bridge + No-Roll-Over Formal Loans'],...
       ['e1 = ' num2str(e1)...
        ', e2 = ' num2str(e2) ...
           e3 = ' num2str(e3) ', one period no-rollover loan r = ' num2str(0)],...
```

```
['c=3 is perfect smoothing']});
xlabel(x_title_line);
ylabel('Consumption');
legend({['consumption at t=1'],...
['consumption at t=2 and t=3']...
}, 'Location', 'east')
% grids
grid on;
grid minor;
```

