Bridge Loan, Uncertainty and Risky Investment

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In Multi-period Loans, One Period Loans, Roll-over Loans, Bridge Loans, we defined what multi-period, one-period, roll-over and bridge loans are in our models. We discussed the role of bridge loan in improving consumption smoothing when only one period no-roll-over loans are allowed in a 3 period deterministic model with fixed but unevenly distributed endowments.

In the setting below, we discuss the role of bridge-loan in a dynamic investment context with uncertainty.

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A Three Period Model With Uncertainty and Investment

There are three model periods, and two possible states.

- Households can borrow/save in the first period to finance either consumption, c_1, or capital investments, K.
- Households can also borrow/save in the second period given shock realization.
- initial period: $e_1 = 0$
- low state: $Y_{t,l} = 1$, p_l is the low state probability
- high state: $Y_{t,h} = A \cdot K + 1$, $p_h = 1 - p_l$ is the high state probability
- The two shocks are i.i.d.

In the first period, households choose K and b_2. In the second period, conditional on realized shocks and b_2, households choose b_3. The first period's problem is:

$$\max_{K \geq 0, b_2} \left\{ \log(c_1) + \sum_{s \in \{l,h\}} p_s \cdot \log(c_{2,s}) + \sum_{s \in \{l,h\}} \sum_{\hat{s} \in \{l,h\}} p_s \cdot p_{\hat{s}} \cdot \log(c_{3,s}) \right\}$$

Not allowing ***bridge loans*** for now, the budget constraints are:

$$c_1 + b_2 + K = e_1$$

$$c_2 + b_3(b_2, s) = Y_{2,s}(K) + b_2$$

and, $c_3 = Y_{3,s}(K) + b_3(b_2, s)$

where $b_3(b_2, s)$ is the second period optimal choice function

If ***roll-over loans*** are allowed, then the implicit (*but never-binding*) borrowing constraint is the *natural borrowing constraint* determined by endowment in the lower state of shock--this is to gurantee repayments even if the household gets hit with two low shocks in a row:

$$b_2 > -(Y_{2,l} + Y_{3,l})$$

and, $b_3(b_2, s) > -(Y_{3,l})$

On the other hand, following the previous discussions, if households only have ***one-period no-roll-over loans***, then the borrowing constraint (*possibly binding*) is based only on endowment in the next period, this is tighter than the *natual borrowing constraint*:

$$b_2 \geq -Y_{2,l}$$

and $b_3(b_2, s) \geq -Y_{3,l}$

Is Bridge Loan Needed If $A = 0$? No

If capital investment does not lead to additional outputs in the high state, households face a fixed endowment of 1 in all future states, uncertainty becomes irrelevant, and we are back to the model in Multi-period Loans, One Period Loans, Roll-over Loans, Bridge Loans.

$$\max_{b_2, b_3} \log(c_1) + \log(c_2) + \log(c_3)$$

such that: $c_1 + b_2 = e_1$, $c_2 + b_3 = e_2 + b_2$, and $c_3 = e_3 + b_3$

Previously, the endowment stream was significantly increasing: $\mathbf{e} = \{e_1 = 0, e_2 = 1, e_3 = 8\}$, and that gave bridge loan a role to complement short-term no-roll-over loans. Now the endowment stream is: $\mathbf{e} = \{e_1 = 0, e_2 = 1, e_3 = 1\}$, optimal choices are:

$$b_2^* = -\frac{2}{3}, b_3^* = -\frac{1}{3}$$

$$c_1^* = c_2^* = c_3^* = \frac{0 + 1 + 1}{3} = \frac{2}{3}$$

Bridge Loan is not needed in this context even with *no-roll-over* loans, because the no-roll-over borrowing constraint does not bind: $b_2^* > -e_2$, optimal borrowing choice is less than 2nd period endowment.

$A \neq 0$ and Optimal Capital Choice with Roll-Over Loans

Now we solve for the optimal borrowing choices when roll-overs are allowed and $A \neq 0$, which makes shock matter, and increases households desire for borrowing to not only smooth consumption but also increase income through capital investments.

Investment and Borrowing:

- With $A = 3$, and $p_l = 0.5$, optimal choices are:

$$K^* = 0.68 \text{ and } b_2^* = -1.35$$

- Second period b_3 choice, **Borrow** given low shock, **save** given high shock:

$$b_3(b_2^*, s = l) = -0.77,\ b_3(b_2^*, s = h) = 0.09$$

Consumption Paths:

- Consumption in the first period:

$$c_1^* = 0.67$$

- Consumption path with **low** shock in the second period, and another bad shock or good shock in the third period:

$$c_{2,l}^* = 0.42,\ c_{3,l}^*(b_3(b_2^*, s = l)) = 0.23,\ c_{3,h}^*(b_3(b_2^*, s = l)) = 2.28$$

- Consumption path with **high** shock in the second period, and a bad shock or another good shock in the third period:

$$c_{2,h}^* = 1.61,\ c_{3,l}^*(b_3(b_2^*, s = h)) = 1.09,\ c_{3,h}^*(b_3(b_2^*, s = h)) = 3.13$$

Compare to $A = 0$:

1. Borrowing to smooth consumption as well as increasing investments
2. Consumptions are **higher** along the **path** where second period shock is h. And when the third period shock is h.
3. Consumption is **lower** for $c_{2,l}^*$ and $c_{3,l}^*(b_3(b_2^*, s = l))$, where households have to pay back debts but have bad shock.

We obtain the above solutions following the codes below:

```
% Some Functions
% Normally, second period problem is solved first than first
% Given we have only 2 states, we can solve them jointly, with b3Hs b3Ls
syms As Ks ps e1s e2s e3s b2s b3LsT2 b3HsT2 b3Ls b3Hs
expected_t2_income = ps*e2s + (1-ps)*(e2s + As*Ks);
expected_t3_income = ps*e3s + (1-ps)*(e3s + As*Ks);
expected_utility = log(e1s-b2s-Ks) + ...
                   ps*log(e2s + b2s - b3LsT2) + ...
                   (1-ps)*log(As*Ks + e2s + b2s - b3HsT2) + ...
                   (ps)*(ps)*log(e3s + b3Ls) +...
                   (ps)*(1-ps)*log(As*Ks + e3s + b3Ls) +...
```

```
                    (1-ps)*(ps)*log(e3s + b3Hs) +...
                    (1-ps)*(1-ps)*log(As*Ks + e3s + b3Hs);

% Variables and Parmaeters
e1=0;
e2=1;
e3=1;
p=0.50;
A=3;

% Function Handle for Constrained Maximization Problem
% x(1) is K
% x(2) is b2
% x(3) is b3L
% x(4) is b3H
U_neg = @(x) (-1)*(log(e1-x(2)-x(1)) + ...
        p*log(e2 + x(2) - x(3)) + ...
        (1-p)*log(A*x(1) + e2 + x(2) - x(4)) + ...
        (p)*(p)*log(e3 + x(3)) +...
        (p)*(1-p)*log(A*x(1) + e3 + x(3)) +...
        (1-p)*(p)*log(e3 + x(4)) +...
        (1-p)*(1-p)*log(A*x(1) + e3 + x(4)));

% Linear Inequality Constraints K>=0, and b2, b3L and b3H can be positive or negative
LIN_CON = [-1,0,0,0];
q = [0];

% Starting Search Points
b0 = [0.1,-0.15,-0.15,-0.15];

% Optimization
[x_argmin, V_at_argmin] = fmincon(U_neg, b0, LIN_CON, q);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>
```

```
% Optimal Risky and Safe Asset Choices
K_o = x_argmin(1);
b2_o = x_argmin(2);
b3L_o = x_argmin(3);
b3H_o = x_argmin(4);

% Value at optimal Choices and Expected Income
EY = double(subs(expected_t2_income, {e2s, ps, As, Ks}, {e2, p, A, K_o}));
EV = double(subs(expected_utility,...
              {As, Ks, ps, e1s, e2s, e3s, b2s, b3LsT2, b3HsT2, b3Ls, b3Hs},...
              {A, K_o, p, e1, e2, e3, b2_o, b3L_o, b3H_o, b3L_o, b3H_o}));

% Consumption
```

```
c1_o = e1-b2_o-K_o;
c2_l_o = e2 + b2_o - b3L_o;
c2_h_o = e2 + A*K_o + b2_o - b3H_o;
c3_ll_o = e3 + b3L_o;
c3_lh_o = e3 + A*K_o + b3L_o;
c3_hl_o = e3 + b3H_o;
c3_hh_o = e3 + A*K_o + b3H_o;

% Display Results
b3L_B_o = false;
b3H_B_o = false;
opti_K_b2_b3 = table(K_o, b2_o, b3L_o, b3H_o, b3L_B_o, b3H_B_o, EY, EV);
opti_consumption = table(K_o, c1_o,...
                         c2_l_o, c3_ll_o, c3_lh_o,...
                         c2_h_o, c3_hl_o, c3_hh_o);
opti_K_b2_b3.Variables =  round(opti_K_b2_b3.Variables, 2);
disp(opti_K_b2_b3);
```

K_o	b2_o	b3L_o	b3H_o	b3L_B_o	b3H_B_o	EY	EV
0.68	-1.35	-0.77	0.09	false	false	2.02	-0.45

```
opti_consumption.Variables =  round(opti_consumption.Variables, 2);
disp(opti_consumption);
```

K_o	c1_o	c2_l_o	c3_ll_o	c3_lh_o	c2_h_o	c3_hl_o	c3_hh_o
0.68	0.67	0.42	0.23	2.28	1.61	1.09	3.13

$A \neq 0$ and Optimal Capital Choice with No-Roll-Over Loan

Above, we solved the maximization problem with loans that allowed for roll-over. What are the optimal choices and consumption paths when roll-over loans are not allowed?

To answer that, we resolve the problem above, but now imposeg an additional borrowing constraint for first period borrowing choice, our one-period no-roll-over constraint:

$$b_2 \geq -Y_{2,l}$$

Compared to the world where roll-over loans are allowed, the optimal choices show that:

- There is less capital investments, and less expected income, and lower welfare overall.
- Consumption in the high shock periods as well as the initial period are much lower.
- Consumption in the second period with l shock, and in the third period after two l shocks are higher because debt burdens are lower.

```
% Linear Inequality Constraints -K<=0, b2>-e2, and b3L and b3H can be positive or negative
LIN_CON_no_roll_over = [-1, 0,0,0;
                         0,-1,0,0];
q_no_roll_over = [0;e2];
```

```
% Optimization
[x_argmin, V_at_argmin] = fmincon(U_neg, b0, LIN_CON_no_roll_over, q_no_roll_over);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>
```

```
% Optimal Risky and Safe Asset Choices
K_o = x_argmin(1);
b2_o = x_argmin(2);
b3L_o = x_argmin(3);
b3H_o = x_argmin(4);

% Value at optimal Choices and Expected Income
EY = double(subs(expected_t2_income, {e2s, ps, As, Ks}, {e2, p, A, K_o}));
EV = double(subs(expected_utility,...
                          {As, Ks, ps, e1s, e2s, e3s, b2s, b3LsT2, b3HsT2, b3Ls, b3Hs},...
                          {A, K_o, p, e1, e2, e3, b2_o, b3L_o, b3H_o, b3L_o, b3H_o}));

% Consumption
c1_o = e1-b2_o-K_o;
c2_l_o = e2 + b2_o - b3L_o;
c2_h_o = e2 + A*K_o + b2_o - b3H_o;
c3_ll_o = e3 + b3L_o;
c3_lh_o = e3 + A*K_o + b3L_o;
c3_hl_o = e3 + b3H_o;
c3_hh_o = e3 + A*K_o + b3H_o;

% Combine Results
b3L_B_o = false;
b3H_B_o = false;
opti_K_b2_b3_nro = table(K_o, b2_o, b3L_o, b3H_o, b3L_B_o, b3H_B_o, EY, EV);
opti_consumption_nro = table(K_o, c1_o,...
                         c2_l_o, c3_ll_o, c3_lh_o,...
                         c2_h_o, c3_hl_o, c3_hh_o);

% No-Roll-Over and Roll-Over Resutls Together
opti_K_b2_b3_all = [opti_K_b2_b3; opti_K_b2_b3_nro];
opti_consumption_all = [opti_consumption; opti_consumption_nro];
opti_K_b2_b3_all.Properties.RowNames = {'roll-over allowed', 'no roll-over'};
opti_consumption_all.Properties.RowNames = {'roll-over allowed', 'no roll-over'};

% Display
opti_K_b2_b3_all.Variables =  round(opti_K_b2_b3_all.Variables, 2);
disp(opti_K_b2_b3_all);
```

```
                    K_o     b2_o     b3L_o     b3H_o     b3L_B_o     b3H_B_o     EY      EV
                    ____    _____    _____     _____     _______     _______    ____    _____
```

roll-over allowed	0.68	-1.35	-0.77	0.09	false	false	2.02	-0.45
no roll-over	0.45	-1	-0.62	-0.02	false	false	1.68	-0.58

```
opti_consumption_all.Variables =  round(opti_consumption_all.Variables, 2);
disp(opti_consumption_all);
```

	K_o	c1_o	c2_l_o	c3_ll_o	c3_lh_o	c2_h_o	c3_hl_o	c3_hh_o
roll-over allowed	0.68	0.67	0.42	0.23	2.28	1.61	1.09	3.13
no roll-over	0.45	0.55	0.62	0.38	1.74	1.38	0.98	2.35

Budget and Constraints with No-Roll-Over Loans + Bridge Loans

To improve the sitution given no-roll-over one-period loans, we can introduce bridge loans.

Following the discussions in Multi-period Loans, One Period Loans, Roll-over Loans, Bridge Loans, with bridge loans, the budget and constraints become:

$$c_1 + b_2^A + K = e_1$$

$$c_2 + b_3^A\left(b_2^A, s\right) + b_3^B\left(b_2^A, s\right) = Y_{2,s}(K) + b_2^A$$

$$c_3 = Y_{3,s}(K) + b_3^A\left(b_2^A, s\right) + b_3^B\left(b_2^A, s\right) \cdot (1 + r_{\text{B}})$$

And the borrowing constraints are:

debt is less than endowment and new B borrowing: $b_2^A \geq -Y_{2,s}(K) + b_3^B\left(b_2^A, s\right)$

and $b_3^A\left(b_2^A, s\right) + b_3^B\left(b_2^A, s\right) \cdot (1 + r_{\text{B}}) \geq -Y_{3,s}(K)$

We have to solve for the optimal bridge loan choice in both h and l states in the second period.

Optimal Investments and Borrowing given Bridge Loan + No-Roll-Over Loan

Given $r_B = 0.30$, we can solve for optimal (informal) bridge loan borrowing quantity in addition to (formal) no-roll-over one period loans. We just have to change our linear constraint slightly:

To solve the updated problem:

1. The linear constraint matrix needs two more column for the bridge loan choice for h and l: $b_3^B\left(b_2^A, s\right)$
2. Two additional negativity constraint rows for the bridge loan choices for h and l: $b_3^B\left(b_2^A, s\right) \leq 0$
3. Two updated borrowing constraint for b_2^A: $b_2^A - b_3^B\left(b_2^A, l\right) \geq -Y_{2,l}$, and $b_2^A - b_3^B\left(b_2^A, l\right) + A \cdot K \geq -Y_{2,l}$

The optimal choices, shown in table form below indicates that investment given bridge + no-roll-over is between no-roll-over and roll-over worlds.

```
% Bridge Loan Interest Rate
rB = 0.30;

% Linear Inequality Constraints
```

```
% Columns 1 to 4 are for:
% 1:K>0 --> -K < 0
% 2:b2 - bB(3,l) >= -e(2,l) --> e(2,l) >= -b2 + bB(3,l)
% 3:b2 - bB(3,l) >= -e(2,l) - A*K --> e(2,l) >= -b2 + bB(3,l) - AK
% 4:bridge low < 0
% 5:bridge high < 0
LIN_CON_no_rlov_brdge = [-1, 0, 0, 0, 0, 0;
                          0,-1, 0, 0, 1, 0;
                         -A,-1, 0, 0, 0, 1;
                          0, 0, 0, 0, 1, 0;
                          0, 0, 0, 0, 0, 1;];
q_no_rlov_brdge = [0;e2;e2;0;0];

% Starting Search Points
% K, b2, b3L, b3H, b3Lbridge, b3Hbridge
b0_no_rlov_brdge = [0.1,-0.15,-0.15,-0.15,-0.01,-0.01];

% Add the Bridge Loan Choice to the Utility
U_neg = @(x) (-1)*(log(e1-x(2)-x(1)) + ...
        p*log(e2 + x(2) - x(3) - x(5)) + ...
        (1-p)*log(A*x(1) + e2 + x(2) - x(4) - x(6)) + ...
        (p)*(p)*log(e3 + x(3) + x(5)*(1+rB)) +...
        (p)*(1-p)*log(A*x(1) + e3 + x(3) + x(5)*(1+rB)) +...
        (1-p)*(p)*log(e3 + x(4) + x(6)*(1+rB)) +...
        (1-p)*(1-p)*log(A*x(1) + e3 + x(4) + x(6)*(1+rB)));

% Optimization
[x_argmin, V_at_argmin] = fmincon(U_neg,...
    b0_no_rlov_brdge, LIN_CON_no_rlov_brdge, q_no_rlov_brdge);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>
```

```
% Optimal Risky and Safe Asset Choices
K_o = x_argmin(1);
b2_o = x_argmin(2);
b3L_o = x_argmin(3);
b3H_o = x_argmin(4);
b3L_B_o = x_argmin(5); % Bridge bad shock t = 2
b3H_B_o = x_argmin(6); % Bridge good shock t = 2

% Value at optimal Choices and Expected Income
EY = double(subs(expected_t2_income, {e2s, ps, As, Ks}, {e2, p, A, K_o}));
EV = double(subs(expected_utility,...
                          {As, Ks, ps, e1s, e2s, e3s, b2s,...
                           b3LsT2, b3HsT2,...
                           b3Ls, b3Hs},...
                          {A, K_o, p, e1, e2, e3, b2_o,...
                           (b3L_o+b3L_B_o), (b3H_o+b3H_B_o),...
```

```
                            (b3L_o+b3L_B_o*(1+rB)), (b3H_o+b3H_B_o*(1+rB))}));

% Consumption
c1_o = e1-b2_o-K_o;
c2_l_o = e2 + b2_o - b3L_o - b3L_B_o;
c2_h_o = e2 + A*K_o + b2_o - b3H_o  - b3H_B_o;
c3_ll_o = e3 + b3L_o + b3L_B_o*(1+rB);
c3_lh_o = e3 + A*K_o + b3L_o + b3L_B_o*(1+rB);
c3_hl_o = e3 + b3H_o + b3H_B_o*(1+rB);
c3_hh_o = e3 + A*K_o + b3H_o + b3H_B_o*(1+rB);

% Combine Results
opti_K_b2_b3_nro_brdg = table(K_o, b2_o, b3L_o, b3H_o, b3L_B_o, b3H_B_o, EY, EV);
opti_K_b2_b3_nro_brdg.Variables =  round(opti_K_b2_b3_nro_brdg.Variables, 2);
disp(opti_K_b2_b3_nro_brdg);
```

K_o	b2_o	b3L_o	b3H_o	b3L_B_o	b3H_B_o	EY	EV
0.57	-1.17	-0.49	0.03	-0.17	0	1.85	-0.54

```
opti_consumption_nro_brdg = table(K_o, c1_o,...
                                  c2_l_o, c3_ll_o, c3_lh_o,...
                                  c2_h_o, c3_hl_o, c3_hh_o);

% No-Roll-Over and Roll-Over Resutls Together
opti_K_b2_b3_all = [opti_K_b2_b3; opti_K_b2_b3_nro; opti_K_b2_b3_nro_brdg];
opti_K_b2_b3_all.Properties.RowNames = {'roll-over allowed',...
                                        'no roll-over',...
                                        ['no roll-over + bridge r=' num2str(rB)]};

opti_consumption_all = [opti_consumption; opti_consumption_nro; opti_consumption_nro_brdg];
opti_consumption_all.Properties.RowNames = {'roll-over allowed',...
                                        'no roll-over',...
                                        ['no roll-over + bridge r=' num2str(rB)]};

% Display
opti_K_b2_b3_all.Variables =  round(opti_K_b2_b3_all.Variables, 2);
disp(opti_K_b2_b3_all);
```

	K_o	b2_o	b3L_o	b3H_o	b3L_B_o	b3H_B_o	EY	EV
roll-over allowed	0.68	-1.35	-0.77	0.09	0	0	2.02	-0.45
no roll-over	0.45	-1	-0.62	-0.02	0	0	1.68	-0.58
no roll-over + bridge r=0.3	0.57	-1.17	-0.49	0.03	-0.17	0	1.85	-0.54

```
opti_consumption_all.Variables =  round(opti_consumption_all.Variables, 2);
disp(opti_consumption_all);
```

	K_o	c1_o	c2_l_o	c3_ll_o	c3_lh_o	c2_h_o	c3_hl_o	c3_hh_o
roll-over allowed	0.68	0.67	0.42	0.23	2.28	1.61	1.09	3.13
no roll-over	0.45	0.55	0.62	0.38	1.74	1.38	0.98	2.35

no roll-over + bridge r=0.3	0.57	0.61	0.49	0.28	1.99	1.5	1.03	2.74

Optimal Investments and Consumptions with Shifting Bridge Loan Interest Rates

Keep the same parameters as before, we will now solve for optimal choices given different bridge loan interest rates.

```
% Loop over variations in interest rates of the bridge loan
bridge_r_min = 0.01; % no better than main/formal loan r
bridge_r_max = 1.0; % if higher than this, want to save
bridge_r_n = 100;
bridge_loan_r_vector = linspace(bridge_r_min, bridge_r_max, bridge_r_n);

lows_p_min = 0.40;
lows_p_max = 0.60;
lows_p_n = 3;
lows_p_vector = linspace(lows_p_min, lows_p_max, lows_p_n);

% Matrixes to Store Relevant Values
value_mat = zeros(lows_p_n, bridge_r_n);
K_o_mat = zeros(lows_p_n, bridge_r_n);
c2ll_o_mat = zeros(lows_p_n, bridge_r_n);
b3LB_frac_o_mat = zeros(lows_p_n, bridge_r_n);

% Table Storage
table_results_all = [];
table_rows_all = {};
graph_counter_all = 0;

% Loop over bridge loan
for lows_p_ctr=1:length(lows_p_vector)
    p = lows_p_vector(lows_p_ctr);
    for bridge_r_ctr=1:length(bridge_loan_r_vector)
        rB = bridge_loan_r_vector(bridge_r_ctr);

        % Bridge Loan utility function with varying interest rates
        U_neg = @(x) (-1)*(log(e1-x(2)-x(1)) + ...
                p*log(e2 + x(2) - x(3) - x(5)) + ...
                (1-p)*log(A*x(1) + e2 + x(2) - x(4) - x(6)) + ...
                (p)*(p)*log(e3 + x(3) + x(5)*(1+rB)) +...
                (p)*(1-p)*log(A*x(1) + e3 + x(3) + x(5)*(1+rB)) +...
                (1-p)*(p)*log(e3 + x(4) + x(6)*(1+rB)) +...
                (1-p)*(1-p)*log(A*x(1) + e3 + x(4) + x(6)*(1+rB)));

        % Optimization
        options = optimoptions('FMINCON','Display','off');
        [x_argmin, V_at_argmin] = fmincon(U_neg,...
            b0_no_rlov_brdge, LIN_CON_no_rlov_brdge, q_no_rlov_brdge,...
            [], [], [], [], [], options);

        % Optimal Risky and Safe Asset Choices
        K_o = x_argmin(1);
```

```
        b2_o = x_argmin(2);
        b3L_o = x_argmin(3);
        b3H_o = x_argmin(4);
        b3L_B_o = x_argmin(5); % Bridge bad shock t = 2
        b3H_B_o = x_argmin(6); % Bridge good shock t = 2

        % Value at optimal Choices and Expected Income
        EY = double(subs(expected_t2_income, {e2s, ps, As, Ks}, {e2, p, A, K_o}));
        EV = double(subs(expected_utility,...
                                  {As, Ks, ps, e1s, e2s, e3s, b2s,...
                                   b3LsT2, b3HsT2,...
                                   b3Ls, b3Hs},...
                                  {A, K_o, p, e1, e2, e3, b2_o,...
                                   (b3L_o+b3L_B_o), (b3H_o+b3H_B_o),...
                                   (b3L_o+b3L_B_o*(1+rB)), (b3H_o+b3H_B_o*(1+rB))}));

        % Consumption
        c1_o = e1-b2_o-K_o;
        c2_l_o = e2      + b2_o - b3L_o - b3L_B_o;
        c2_h_o = e2 + A*K_o + b2_o - b3H_o  - b3H_B_o;
        c3_ll_o = e3 + b3L_o + b3L_B_o*(1+rB);
        c3_lh_o = e3 + A*K_o + b3L_o + b3L_B_o*(1+rB);
        c3_hl_o = e3 + b3H_o + b3H_B_o*(1+rB);
        c3_hh_o = e3 + A*K_o + b3H_o + b3H_B_o*(1+rB);

        % Store Results
        value_mat(lows_p_ctr, bridge_r_ctr) = EV;
        c2ll_o_mat(lows_p_ctr, bridge_r_ctr) = c3_ll_o;
        K_o_mat(lows_p_ctr, bridge_r_ctr) = K_o;
        b3LB_frac_o_mat(lows_p_ctr, bridge_r_ctr) = b3L_B_o/b2_o;

        % Combine Results
        opti_K_b2_b3_nro_brdg = table(K_o, b2_o, b3L_o, b3H_o, b3L_B_o, b3H_B_o, EY, EV);

        % Store to Table
        graph_counter_all = graph_counter_all + 1;
        table_results_all = [table_results_all; opti_K_b2_b3_nro_brdg];
        table_row_names_all{graph_counter_all} =...
            ['brdg-r:' sprintf('%3.2f', rB) ' low-p:' sprintf('%3.2f', p)];

    end
end
```

Graphing the Effects of Bridge Loans with Varying Interest Rates

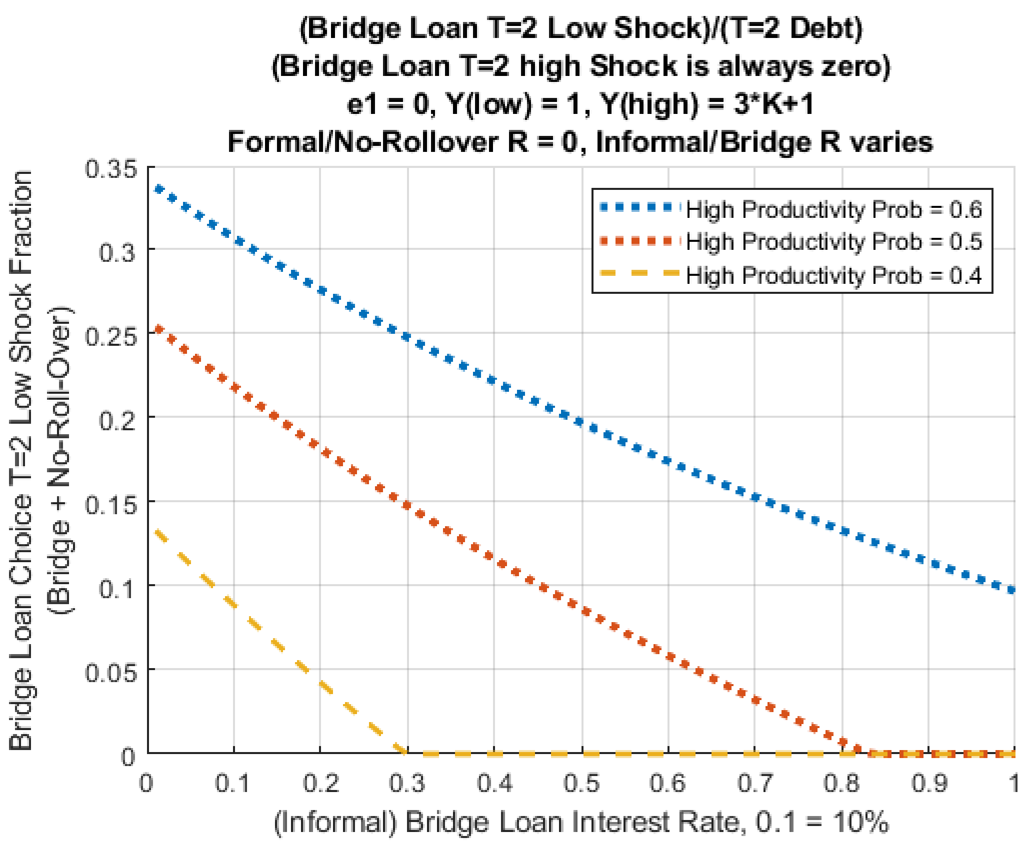
Similar to what we saw before, bridge loan has significant effects on all model outcomes. Bridge loans are useful even when bridge loan interest rates are close to 100 percent:

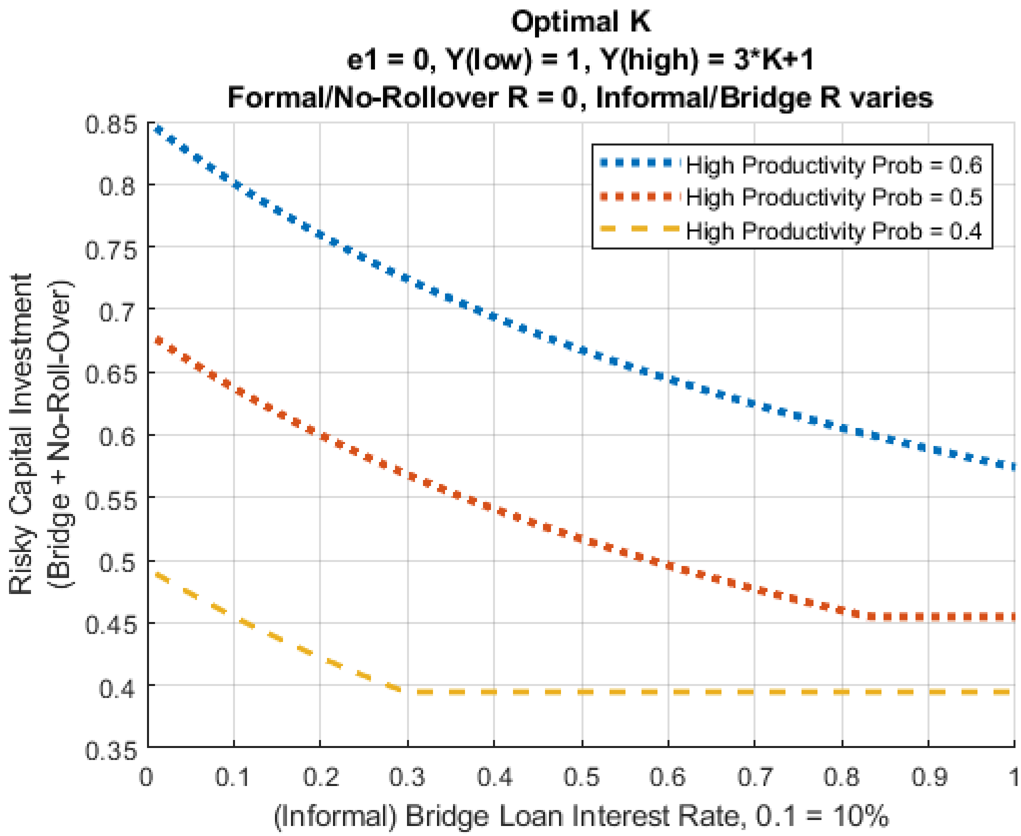
- If the chance of the high productivity state happening is higher, bridge loan demand is higher.
- Bridge loan usage is higher when its rates are lower
- Bridge loans help to signfiicantly increase optimal capital investments, especially when high productivity probability is higher.

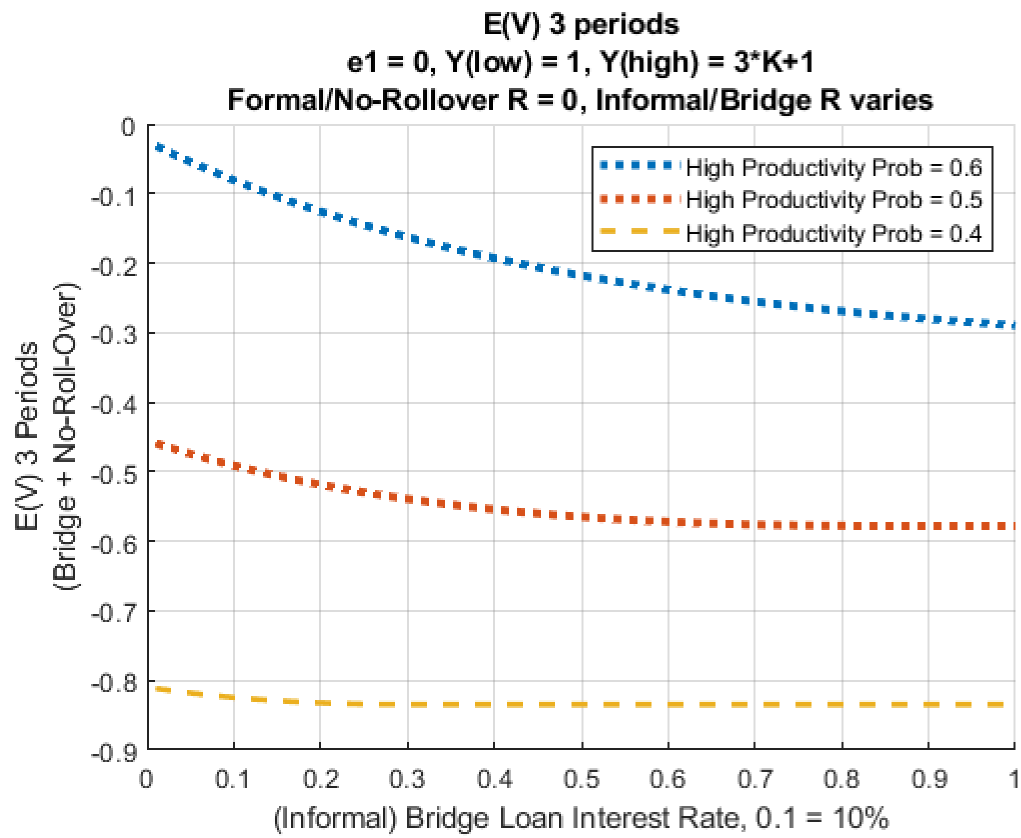
- When investments increase (with lower bridge loan costs), due to higher borrowing initially, consumption along low shock path is worse.

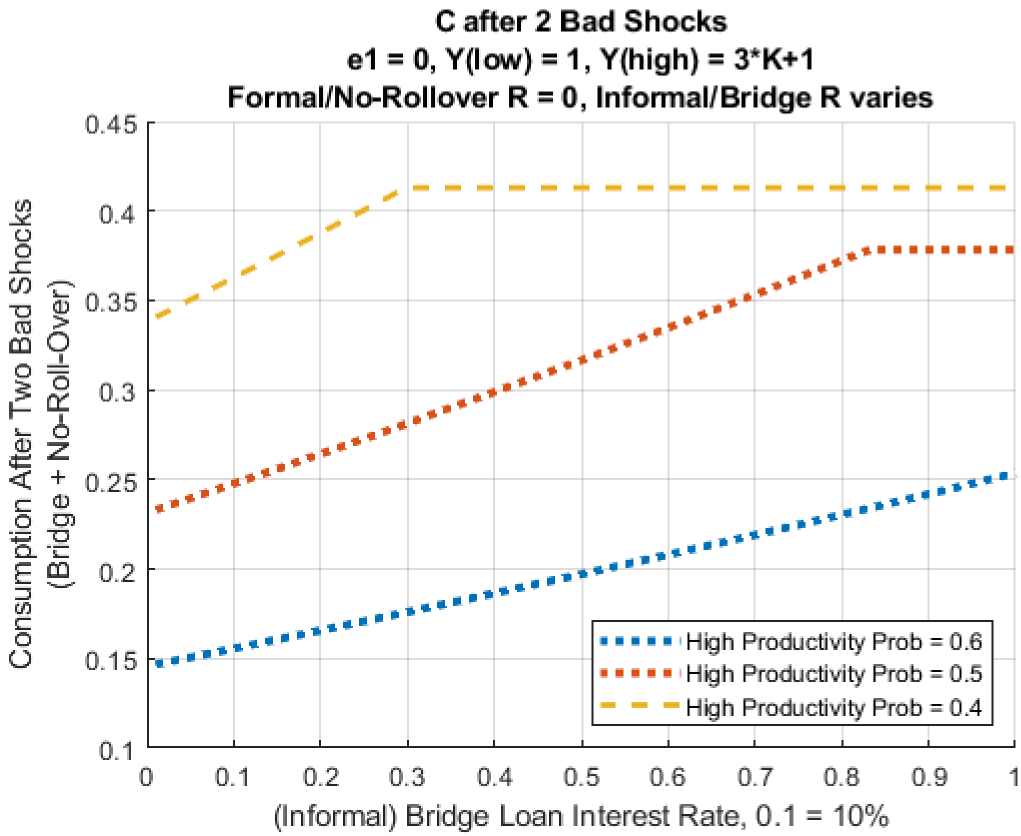
```
for graph_vars=1:1:4

    figure();
    hold on;
    p_legend = {};
    for lows_p_ctr=1:length(lows_p_vector)

        p = lows_p_vector(lows_p_ctr);
        if (graph_vars == 1)
            data_vector = b3LB_frac_o_mat(lows_p_ctr,:);
        elseif (graph_vars == 2)
            data_vector = K_o_mat(lows_p_ctr,:);
        elseif (graph_vars == 3)
            data_vector = value_mat(lows_p_ctr,:);
        elseif (graph_vars == 4)
            data_vector = c2ll_o_mat(lows_p_ctr,:);
        end

        p_legend{lows_p_ctr} = ['High Productivity Prob = ' num2str(1-p)];

        if (lows_p_ctr == 1)
            plot(bridge_loan_r_vector, data_vector, ':', 'LineWidth', 3);
        elseif (lows_p_ctr == 2)
            plot(bridge_loan_r_vector, data_vector, ':', 'LineWidth', 3);
        elseif (lows_p_ctr == 3)
            plot(bridge_loan_r_vector, data_vector, '--', 'LineWidth', 2);
        elseif (lows_p_ctr == 4)
            plot(bridge_loan_r_vector, data_vector, '-',  'LineWidth', 1);
        end

    end

    if (graph_vars == 1)
        title({['(Bridge Loan T=2 Low Shock)/(T=2 Debt)'],...
               ['(Bridge Loan T=2 high Shock is always zero)'],...
               ['e1 = ' num2str(e1)...
                ', Y(low) = ' num2str(e2)...
                ', Y(high) = ' num2str(A) '*K+' num2str(e2)],...
               ['Formal/No-Rollover R = ' num2str(0), ', Informal/Bridge R varies']});
        ylabel({['Bridge Loan Choice T=2 Low Shock Fraction'],['(Bridge + No-Roll-Over)']});
        legend(p_legend, 'Location','northeast');
    elseif (graph_vars == 2)
        title({['Optimal K'],...
               ['e1 = ' num2str(e1)...
                ', Y(low) = ' num2str(e2)...
                ', Y(high) = ' num2str(A) '*K+' num2str(e2)],...
               ['Formal/No-Rollover R = ' num2str(0), ', Informal/Bridge R varies']});
        ylabel({['Risky Capital Investment'],['(Bridge + No-Roll-Over)']});
        legend(p_legend, 'Location','northeast');
```

```
    elseif (graph_vars == 3)
        title({['E(V) 3 periods'],...
               ['e1 = ' num2str(e1)...
                ', Y(low) = ' num2str(e2)...
                ', Y(high) = ' num2str(A) '*K+' num2str(e2)],...
               ['Formal/No-Rollover R = ' num2str(0), ', Informal/Bridge R varies']});
        ylabel({['E(V) 3 Periods'],['(Bridge + No-Roll-Over)']});
        legend(p_legend, 'Location','northeast');
    elseif (graph_vars == 4)
        title({['C after 2 Bad Shocks'],...
               ['e1 = ' num2str(e1)...
                ', Y(low) = ' num2str(e2)...
                ', Y(high) = ' num2str(A) '*K+' num2str(e2)],...
               ['Formal/No-Rollover R = ' num2str(0), ', Informal/Bridge R varies']});
        ylabel('Consumption After Two Bad Shocks');
        ylabel({['Consumption After Two Bad Shocks'],['(Bridge + No-Roll-Over)']});
        legend(p_legend, 'Location','southeast');
    end

    x_title_line = ['(Informal) Bridge Loan Interest Rate, 0.1 = 10%'];
    xlabel(x_title_line);

    % grids
    grid on;
end
```




Optimal K
e1 = 0, Y(low) = 1, Y(high) = 3*K+1
Formal/No-Rollover R = 0, Informal/Bridge R varies
Risky Capital Investment
(Bridge + No-Roll-Over)
(Informal) Bridge Loan Interest Rate, 0.1 = 10%
High Productivity Prob = 0.6
High Productivity Prob = 0.5
High Productivity Prob = 0.4


E(V) 3 periods
e1 = 0, Y(low) = 1, Y(high) = 3*K+1
Formal/No-Rollover R = 0, Informal/Bridge R varies
E(V) 3 Periods
(Bridge + No-Roll-Over)
(Informal) Bridge Loan Interest Rate, 0.1 = 10%
High Productivity Prob = 0.6
High Productivity Prob = 0.5
High Productivity Prob = 0.4



Tabulate All Results

The Tables below tabulate results, showing 100 rows from the graph above:

```
table_results_all.Properties.RowNames = table_row_names_all;
[rows, cols] = size(table_results_all);
% Only show 100 rows
rows_display = floor(linspace(1, rows, 100));
```

8 Columns of Choices and Outcomes in the Table Below:

1. **K_o**: K^*, Optimal capital choice (made in t=1)
2. **b2_o**: $b_2^{A,*}$, Optimal 1st period ***No-Roll-Over*** (formal) borrowing/savings choice (debt to be paid back in t=2). Households will borrow because $e_1 = 0$
3. **b3L_o**: $b_3^{A,*}(b_2^A, l)$, Optimal 2nd period, when shock is l, ***No-Roll-Over*** (formal) borrowing/savings choice (to be paid back in t=3). Households will borrow here because shock is low.
4. **b3H_o**: $b_3^{A,*}(b_2^A, h)$, Optimal 2nd period, when shock is h, ***No-Roll-Over*** (formal) borrowing/savings choice (to be paid back in t=3).
5. **b3L_B_o**: $b_3^{B,*}(b_2^A, l)$, Optimal 2nd period, when shock is l, ***Bridge Loan*** (informal) borrowing choice. Used to pay $b_2^{A,*}$ debts when $Y_{2,l}$--in bad shock state--is insufficient to cover $b_2^{A,*}$ fully. Borrowing bridge loan allows households to clear current debt, $b_2^{A,*}$, with formal lender, and borrow new $b_3^{A,*}(b_2^A, l)$ from formal lender again, which is to be paid back in t=3.

6. **b3H_B_o**: $b_3^B\left(b_2^A, h\right)$, Optimal 2nd period, when shock is h, ***Bridge Loan*** (informal) borrowing choice. This will be 0, because in high shock state, there is enough output to pay back debt $b_2^{A,*}$, bridge loan is unnecessary.
7. **EY**: expected income, same in the 2nd and 3rd period because there is a single investment decision in the first period.
8. **EV**: expected value given optimal choices.

```
table_results_all.Variables =  round(table_results_all.Variables, 2);
disp(table_results_all(rows_display,:))
```

	K_o	b2_o	b3L_o	b3H_o	b3L_B_o	b3H_B_o	EY	EV
brdg-r:0.01 low-p:0.40	0.85	-1.51	-0.34	0.12	-0.51	0	2.52	-0.03
brdg-r:0.04 low-p:0.40	0.83	-1.49	-0.35	0.11	-0.49	0	2.49	-0.05
brdg-r:0.07 low-p:0.40	0.82	-1.46	-0.35	0.1	-0.46	0	2.47	-0.07
brdg-r:0.10 low-p:0.40	0.8	-1.44	-0.36	0.1	-0.44	0	2.44	-0.08
brdg-r:0.13 low-p:0.40	0.79	-1.42	-0.36	0.09	-0.42	0	2.42	-0.1
brdg-r:0.16 low-p:0.40	0.78	-1.41	-0.37	0.08	-0.41	0	2.4	-0.11
brdg-r:0.19 low-p:0.40	0.76	-1.39	-0.37	0.08	-0.39	0	2.37	-0.12
brdg-r:0.22 low-p:0.40	0.75	-1.37	-0.38	0.08	-0.37	0	2.35	-0.13
brdg-r:0.25 low-p:0.40	0.74	-1.36	-0.39	0.07	-0.36	0	2.33	-0.15
brdg-r:0.28 low-p:0.40	0.73	-1.34	-0.39	0.07	-0.34	0	2.32	-0.16
brdg-r:0.31 low-p:0.40	0.72	-1.33	-0.4	0.06	-0.33	0	2.3	-0.17
brdg-r:0.34 low-p:0.40	0.71	-1.31	-0.4	0.06	-0.31	0	2.28	-0.18
brdg-r:0.37 low-p:0.40	0.7	-1.3	-0.41	0.05	-0.3	0	2.26	-0.18
brdg-r:0.40 low-p:0.40	0.69	-1.29	-0.42	0.05	-0.29	0	2.25	-0.19
brdg-r:0.43 low-p:0.40	0.69	-1.27	-0.42	0.05	-0.27	0	2.23	-0.2
brdg-r:0.46 low-p:0.40	0.68	-1.26	-0.43	0.04	-0.26	0	2.22	-0.21
brdg-r:0.49 low-p:0.40	0.67	-1.25	-0.43	0.04	-0.25	0	2.21	-0.22
brdg-r:0.52 low-p:0.40	0.66	-1.24	-0.44	0.04	-0.24	0	2.19	-0.22
brdg-r:0.55 low-p:0.40	0.66	-1.23	-0.44	0.03	-0.23	0	2.18	-0.23
brdg-r:0.58 low-p:0.40	0.65	-1.22	-0.45	0.03	-0.22	0	2.17	-0.24
brdg-r:0.61 low-p:0.40	0.64	-1.21	-0.46	0.03	-0.21	0	2.16	-0.24
brdg-r:0.64 low-p:0.40	0.64	-1.2	-0.46	0.03	-0.2	0	2.15	-0.25
brdg-r:0.67 low-p:0.40	0.63	-1.19	-0.47	0.02	-0.19	0	2.13	-0.25
brdg-r:0.70 low-p:0.40	0.62	-1.18	-0.47	0.02	-0.18	0	2.12	-0.26
brdg-r:0.73 low-p:0.40	0.62	-1.17	-0.48	0.02	-0.17	0	2.11	-0.26
brdg-r:0.76 low-p:0.40	0.61	-1.16	-0.49	0.02	-0.16	0	2.1	-0.26
brdg-r:0.79 low-p:0.40	0.61	-1.16	-0.49	0.02	-0.16	0	2.09	-0.27
brdg-r:0.82 low-p:0.40	0.6	-1.15	-0.5	0.01	-0.15	0	2.08	-0.27
brdg-r:0.85 low-p:0.40	0.6	-1.14	-0.5	0.01	-0.14	0	2.08	-0.28
brdg-r:0.88 low-p:0.40	0.59	-1.13	-0.51	0.01	-0.13	0	2.07	-0.28
brdg-r:0.91 low-p:0.40	0.59	-1.13	-0.51	0.01	-0.13	0	2.06	-0.28
brdg-r:0.94 low-p:0.40	0.58	-1.12	-0.52	0.01	-0.12	0	2.05	-0.28
brdg-r:0.97 low-p:0.40	0.58	-1.11	-0.53	0	-0.11	0	2.04	-0.29
brdg-r:1.00 low-p:0.40	0.57	-1.11	-0.53	0	-0.11	0	2.03	-0.29
brdg-r:0.03 low-p:0.50	0.67	-1.33	-0.43	0.08	-0.33	0	2	-0.47
brdg-r:0.06 low-p:0.50	0.65	-1.31	-0.44	0.07	-0.31	0	1.98	-0.48
brdg-r:0.09 low-p:0.50	0.64	-1.29	-0.44	0.07	-0.29	0	1.96	-0.49
brdg-r:0.12 low-p:0.50	0.63	-1.27	-0.45	0.06	-0.27	0	1.94	-0.5
brdg-r:0.15 low-p:0.50	0.62	-1.25	-0.46	0.06	-0.25	0	1.93	-0.51
brdg-r:0.18 low-p:0.50	0.61	-1.23	-0.46	0.05	-0.23	0	1.91	-0.51
brdg-r:0.21 low-p:0.50	0.6	-1.22	-0.47	0.05	-0.22	0	1.9	-0.52
brdg-r:0.24 low-p:0.50	0.59	-1.2	-0.48	0.04	-0.2	0	1.88	-0.53
brdg-r:0.27 low-p:0.50	0.58	-1.19	-0.49	0.04	-0.19	0	1.87	-0.53
brdg-r:0.30 low-p:0.50	0.57	-1.17	-0.49	0.03	-0.17	0	1.85	-0.54
brdg-r:0.33 low-p:0.50	0.56	-1.16	-0.5	0.03	-0.16	0	1.84	-0.54
brdg-r:0.36 low-p:0.50	0.55	-1.15	-0.51	0.03	-0.15	0	1.83	-0.55
brdg-r:0.39 low-p:0.50	0.54	-1.14	-0.52	0.02	-0.14	0	1.82	-0.55

brdg-r:0.42 low-p:0.50	0.54	-1.12	-0.52	0.02	-0.12	0	1.8	-0.56
brdg-r:0.45 low-p:0.50	0.53	-1.11	-0.53	0.01	-0.11	0	1.79	-0.56
brdg-r:0.48 low-p:0.50	0.52	-1.1	-0.54	0.01	-0.1	0	1.78	-0.56
brdg-r:0.52 low-p:0.50	0.51	-1.09	-0.55	0.01	-0.09	0	1.77	-0.57
brdg-r:0.55 low-p:0.50	0.51	-1.08	-0.55	0	-0.08	0	1.76	-0.57
brdg-r:0.58 low-p:0.50	0.5	-1.07	-0.56	0	-0.07	0	1.75	-0.57
brdg-r:0.61 low-p:0.50	0.49	-1.06	-0.57	0	-0.06	0	1.74	-0.57
brdg-r:0.64 low-p:0.50	0.49	-1.05	-0.58	0	-0.05	0	1.73	-0.57
brdg-r:0.67 low-p:0.50	0.48	-1.04	-0.58	-0.01	-0.04	0	1.72	-0.58
brdg-r:0.70 low-p:0.50	0.48	-1.03	-0.59	-0.01	-0.03	0	1.72	-0.58
brdg-r:0.73 low-p:0.50	0.47	-1.03	-0.6	-0.01	-0.03	0	1.71	-0.58
brdg-r:0.76 low-p:0.50	0.47	-1.02	-0.6	-0.01	-0.02	0	1.7	-0.58
brdg-r:0.79 low-p:0.50	0.46	-1.01	-0.61	-0.02	-0.01	0	1.69	-0.58
brdg-r:0.82 low-p:0.50	0.46	-1	-0.62	-0.02	0	0	1.69	-0.58
brdg-r:0.85 low-p:0.50	0.46	-1	-0.62	-0.02	0	0	1.68	-0.58
brdg-r:0.88 low-p:0.50	0.46	-1	-0.62	-0.02	0	0	1.68	-0.58
brdg-r:0.91 low-p:0.50	0.46	-1	-0.62	-0.02	0	0	1.68	-0.58
brdg-r:0.94 low-p:0.50	0.46	-1	-0.62	-0.02	0	0	1.68	-0.58
brdg-r:0.97 low-p:0.50	0.46	-1	-0.62	-0.02	0	0	1.68	-0.58
brdg-r:1.00 low-p:0.50	0.46	-1	-0.62	-0.02	0	0	1.68	-0.58
brdg-r:0.03 low-p:0.60	0.48	-1.14	-0.51	0	-0.14	0	1.58	-0.82
brdg-r:0.06 low-p:0.60	0.47	-1.12	-0.52	-0.01	-0.12	0	1.56	-0.82
brdg-r:0.09 low-p:0.60	0.46	-1.1	-0.53	-0.01	-0.1	0	1.55	-0.82
brdg-r:0.12 low-p:0.60	0.45	-1.09	-0.54	-0.02	-0.09	0	1.54	-0.83
brdg-r:0.15 low-p:0.60	0.44	-1.07	-0.55	-0.02	-0.07	0	1.53	-0.83
brdg-r:0.18 low-p:0.60	0.43	-1.05	-0.55	-0.03	-0.05	0	1.51	-0.83
brdg-r:0.21 low-p:0.60	0.42	-1.04	-0.56	-0.03	-0.04	0	1.5	-0.83
brdg-r:0.24 low-p:0.60	0.41	-1.03	-0.57	-0.04	-0.03	0	1.49	-0.83
brdg-r:0.27 low-p:0.60	0.4	-1.01	-0.58	-0.04	-0.01	0	1.48	-0.83
brdg-r:0.30 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.33 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.36 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.39 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.42 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.45 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.48 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.51 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.54 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.57 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.60 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.63 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.66 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.69 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.72 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.75 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.78 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.81 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.84 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.87 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.90 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.93 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:0.96 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84
brdg-r:1.00 low-p:0.60	0.4	-1	-0.59	-0.04	0	0	1.47	-0.84