The Distribution of the Gender Wage Gap: An Equilibrium Model

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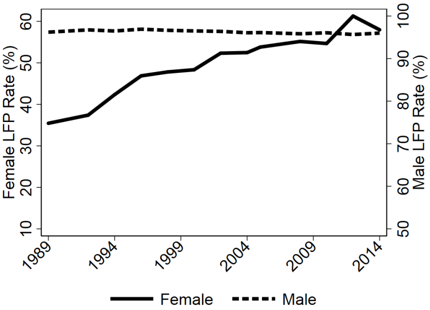
Motivation

- A secular rise in female labor force participation of (FLFP) is one of the most salient features of the labor market over the last century (Killingsworth and Heckman, 1987; Goldin and Olivetti, 2013)
- Mexico 1989-2014: FLFP increased by 50% (35% to 60%)

Changes in the Male and Female LFP

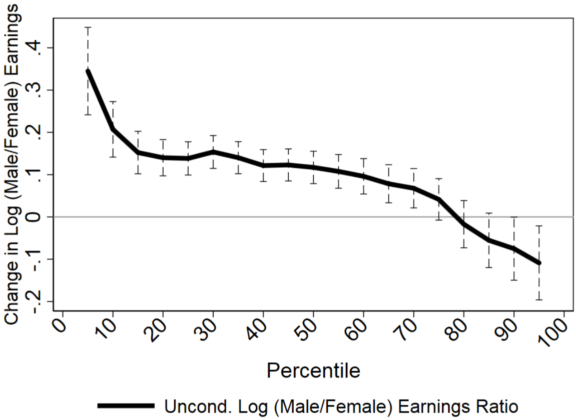


- We are motivated to investigate the impact of this massive change in size and composition of the LF on the gender wage gap

Evolution of the gender wage gap in Mexico

- Our analysis is motivated by dramatic variations in gender wage gap changes across the wage distribution during 1989-2014
- No change at the mean or median
- Widened at the bottom and narrowed at the top
  - below median: widened 10 to 32 percent
  - top quintile: declined 5 to 18 percent
- Our key insight is that these divergent patterns are consistent with an increase in women's labour supply if women **substitute** men more easily in high than in low paying **occupations**

Changes in the Log (Male/Female) Wages Ratio Between C.1992 and C.2012

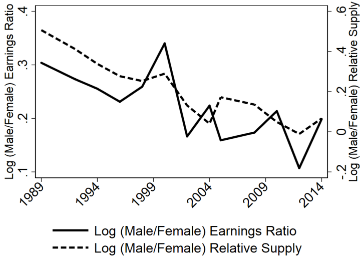


Elasticity of substitution of male vs female labour, varying by task content

- Assume men and women are imperfect substitutes in production
- Increases in women's labor supply will exert downward pressure on male and female wages, but greater downward pressure on female wages, thus widening the gender wage gap
- *How much* will depend upon the elasticity of substitution between male and female labor
- We allow this elasticity to depend on occupational task content (innovation)

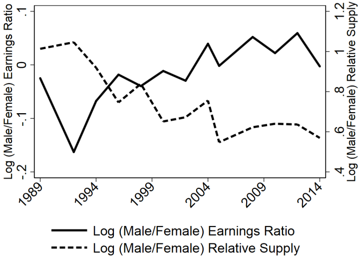
Figure: Comovement in labour supply and wage gaps in routine and manual task-intensive but not abstract-task occupations

(a) Abstract Occupations



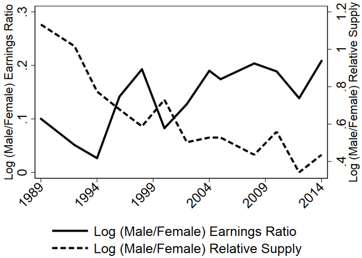
$d\ \ln(L_m/L_f) \approx -44$; $d\ \ln(W_m/W_f) \approx -10$

(b) Routine Occupations



$d\ \ln(L_m/L_f) \approx -44$; $d\ \ln(W_m/W_f) \approx 2.3$

(c) Manual Occupations



Equilibrium model

- We estimate an equilibrium model that extends the canonical labor demand-supply models (Katz and Murphy, 1992; Murphy and Welch, 1992; Katz and Autor, 1999)
  - This produces elasticity and demand parameters
- Demand: nested-CES aggregate production function
  - Elasticities vary by occupation: *abstract*, *routine*, *manual*
  - Time varying labor shares by *gender*, *skill* and *occupation*
- Supply: workers choose an occupation or home production
  - Non-wage supply shifters: i) fertility (<5), ii) marital status, iii) appliance availability, iv) women's rights (WBL)
  - Changing demographics/population by *gender* and *skill* groups

What we find (estimates)

- Elasticity of substitution between male and female labor:
  - $\sigma \approx 2.9$: high-paying abstract task-intensive occupations
  - $\sigma \approx 1.2$: low-paying manual and routine occupations
  - Consistent with $\Uparrow$ FLFP exerting less downward pressure on female relative to male wages in high-paying than in low-paying occupations
- Estimated demand trends:
  - Favored female workers, and more so among the skilled (college educated) in abstract and routine task-intensive occupations
  - Skilled/analytical demand rise overwhelmed supply increases
  - Reverse for low skilled/non-analytical workers/occupations
- The results explain the motivating stylized facts

What we find (counterfactuals)

1. Non-wage determinants of LFP
   - Increasing appliance availability increased unskilled female LFP and hastened the divergence of the gender gap at the bottom of the wage distribution
   - Decreasing fertility for skilled women increased skilled female LFP and muted convergence of the gender wage gap at the top of the wage distribution
2. Demographics
   - The increasing share of skilled women (with college degree) widened the gender LFP gap, and narrowed the wage gap
3. GE can attenuate or magnify PE effects
   - Attenuating non-wage determinants of LFP effects
   - Possibly magnifying demographic effects

Data

Source:

- Mexican Household Income and Expenditure Survey (ENIGH). Nationally representative survey with 13 waves 1989 to 2014.

Income variable:

- Monthly monetary labor remuneration in all occupations.
  Include wages, salaries, piecework, overtime, commissions, or tips.
  Excludes gov. transfers or profits from work in self-employment.
- Converted into **hourly rates**.
- **Full-time workers** (35 hours or more in the previous week).

Sample:

- **Prime-age workers**. Population between the ages of 25 and 55.

Occupation Groups- by task-intensity

ENIGH Principal Group	Median Percentile of the Task Measure: Abstract	Routine	Manual	Group	Av. Share (x100)	Av. Male Share (x100)	Av. Earnings Percentile
Managers	90.0	17.0	27.5	Abstract	2.9	71.3	85.4
Crafts and Trades (Supervisors)	84.0	42.0	62.0	Abstract	1.8	84.2	72.3
Education	83.0	11.0	65.0	Abstract	4.5	38.2	80.2
Professional	83.0	42.0	46.0	Abstract	4.1	62.4	82.3
Technical	71.0	69.0	43.0	Abstract	4.0	59.3	68.6
Arts/Entertainment	66.0	35.0	48.0	Abstract	0.6	76.4	70.4
Sales	61.0	22.5	15.0	Abstract	12.7	46.3	47.5
Crafts and Trades (Laborers)	40.0	82.0	73.0	Routine	14.3	76.4	47.4
Clerical (Supervisors)	61.0	63.0	51.5	Routine	2.5	65.0	77.9
Crafts and Trades (Helpers)	10.5	62.0	60.5	Routine	5.8	80.4	34.8
Machine Operators	16.0	62.0	51.0	Routine	3.6	62.4	48.4
Clerical (Laborers)	41.5	53.0	12.0	Routine	6.6	37.3	60.4
Transport	19.5	21.0	96.0	Manual	5.8	99.0	46.9
Agriculture	32.0	27.0	82.0	Manual	13.2	78.6	20.9
Protective Services	24.5	5.5	76.5	Manual	2.3	93.1	44.4
Domestic Service	9.0	8.0	76.0	Manual	4.1	7.6	27.0
Street Sales	38.0	13.0	64.0	Manual	3.4	44.0	30.3
Service	28.0	25.0	63.0	Manual	7.4	43.4	40.2

Task-Based Approach

First we check whether compositional change drives changes in the wage distribution by gender

$$\ln W_{gen,t} = X'_{gen,t}\beta_{gen,t} + \epsilon_{gen,t}, \quad \text{for} \quad gen = (\text{male}(k), \text{female}(f))$$

- X: all interactions of 7 education, 6 age and 18 occupations.
- Unconditional quantile regressions (Firpo et al. 2009) ▸ Details

▪ Difference over time:

$$\Delta_t \hat{q}_{\tau,gen} = \underbrace{\left(\overline{X'}_{gen,\text{C.2012}} - \overline{X'}_{gen,\text{C.1992}}\right)\hat{\beta}_{gen,P}}_{\Delta_t \hat{q}_{X,\tau,gen}} + \underbrace{\overline{X'}_{gen,P}\left(\hat{\beta}_{gen,\text{C.2012}} - \hat{\beta}_{gen,\text{C.1992}}\right)}_{\Delta_t \hat{q}_{S,\tau,gen}}$$

▪ Difference over time + difference across genders:

$$\underbrace{\Delta_t \hat{q}_{\tau,k} - \Delta_t \hat{q}_{\tau,f}}_{\text{Overall}} = \underbrace{\left(\Delta_t \hat{q}_{X,\tau,k} - \Delta_t \hat{q}_{X,\tau,f}\right)}_{\text{Composition}} + \underbrace{\left(\Delta_t \hat{q}_{S,\tau,k} - \Delta_t \hat{q}_{S,\tau,f}\right)}_{\text{Wage Structure}}$$

Results—compositional changes do not explain changes in the gender wage gap (PE)

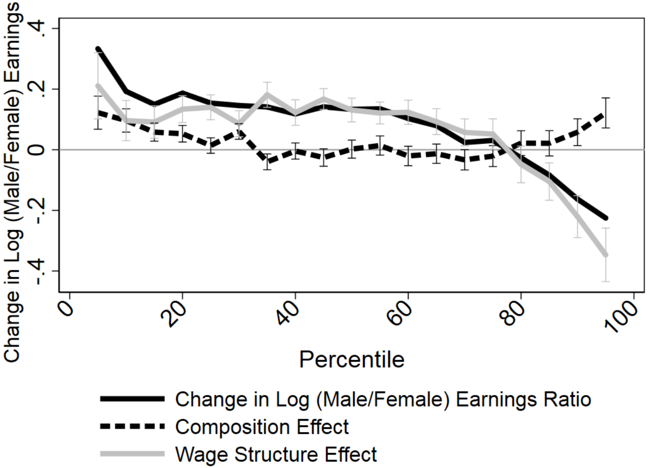


Figure: Decomposition: Change in Log (Male/Female) Earnings Ratio Between C.1992 and C.2012 Attributed to Changes in Composition and Wage Structure

Model: Framework I

Rising FLFP can influence the wage structure through

- Imperfect substituability of M and F labour
- Gender-biased technological change
- Demographic composition changes
- Non-wage shifters of labour supply

In contrast to most of the literature, we model these channels as operating in a context in which labor supply is allowed to respond to changes in the wage structure (which arise from both demand and supply channels in the model)

Model: Framework II

To illustrate, for occupation o (among three), gender gen, at time t ...

$$\textbf{Supply}\text{:}\ L^s_{o,gen,t} = L^{pop}_{gen,t} \cdot F_o(\{\psi \cdot W_{\hat{o},gen,t} + \pi_{\hat{o}} \cdot B_{gen,t}\}^3_{\hat{o}=1})$$

- $L^s_{o,gen,t}$: Occupation- and gender-specific labor supply
- $L^{pop}_{gen,t}$: Gender population level at time t
- $B_{gen,t}$: Vector of observables that shift the relative utility of occ.

$$\textbf{Demand}\ \text{Optimality: } \log\left(\frac{W_{o,k,t}}{W_{o,f,t}}\right) = \log\left(\frac{\alpha_{o,t}}{1-\alpha_{o,t}}\right) - \frac{1}{\sigma_{\rho_o}} \log\left(\frac{L_{o,k,t}}{L_{o,f,t}}\right)$$

- $L_{o,k,t}, W_{o,k,t}$ and $L_{o,f,t}, W_{o,f,t}$: Male and female workers and wages
- $\rho_o \in (-\infty, 1]$: Gender elasticity of substitution: $\sigma_{\rho_o} = \frac{1}{1-\rho_o}$
- $\alpha_{o,t}$: Share parameters (intensity in which labor inputs are used)

Model: Framework III

$$\textbf{Supply}: \ L^s_{o,gen,t} = \underbrace{L^{pop}_{gen,t}}_{\text{Ctr. II}} \cdot F_o(\{\psi \cdot W_{\hat{o},gen,t} + \pi_{\hat{o}} \cdot \underbrace{B_{gen,t}}_{\text{Ctr. I}}\}^3_{\hat{o}=1})$$

$$\textbf{Demand Optimality}: \ \log\left(\frac{W_{o,k,t}}{W_{o,f,t}}\right) = \log\underbrace{\left(\frac{\alpha_{o,t}}{1-\alpha_{o,t}}\right)}_{\text{Ctr. III}} - \frac{1}{\sigma_{\rho_o}}\log\left(\frac{L_{o,k,t}}{L_{o,f,t}}\right)$$

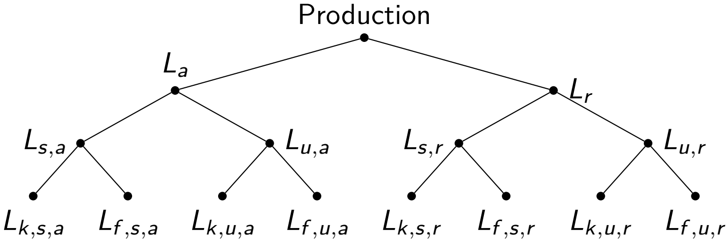
- Exogenous changes:
  1. If $\pi > 0$, supply shifters $B_{gen,t}$ (data) impact LFP
  2. Supply curves shift with compositional changes in $L^{pop}_{gen,t}$ (data)
  3. Relative demands shift with $\alpha_{o,t}$ (estimated)
- $\boldsymbol{W}_t$ and $\boldsymbol{L}_t$ endogenously adjust (resolved in model)
- If $\psi > 0$, non-zero wage elasticity:
  1. $B_{gen,t}$ has "indirect" effects on LFP via wage
  2. $L^{pop}_{gen,t}$ impacts LFP via wage
  3. $\alpha_{o,t}$ impact LFP via wage

Model: Demand

Aggregate production with Three levels of nested CES:

1 **Occupations**: abstract (a), routine (r), and manual (m).
2 **Education**: skilled (s) and unskilled (u).
3 **Gender**: male (k) and female (f).

Branches L_a (abstract), L_r (routine), L_m (manual), two shown:



Model: Demand level 1

Level 1: Task-intensive Occupations.

$$Y_t = Z_t \left[\alpha_{1,t} L_{a,t}^{\rho_1} + (1 - \alpha_{1,t}) \Big(\alpha_{2,t} L_{r,t}^{\rho_2} + (1 - \alpha_{2,t}) L_{m,t}^{\rho_2} \Big)^{\rho_1/\rho_2} \right]^{1/\rho_1}$$

- Y_t: total output
- Z_t: scale parameter (TFP, skill-neutral technological change)
- $L_{a,t}$, $L_{r,t}$, and $L_{m,t}$: total labor in abstract, routine, and manual tasks.
- $\rho_1, \rho_2 \in (-\infty, 1]$: elasticities between *occ* types
- $\alpha_{1,t}$, and $\alpha_{2,t}$: share parameters (demand shifters)

Model: Demand level 2

Level 2: Skills.

$$L_{occ,t} = \left[\alpha_{3,occ,t} L_{s,occ,t}^{\rho_{3,occ}} + (1 - \alpha_{3,occ,t}) L_{u,occ,t}^{\rho_{3,occ}}\right]^{1/\rho_{3,occ}} \quad \text{for} \quad occ = a, r, m,$$

- $L_{occ,t}$: Aggregate occupation-specific labor
- $L_{s,occ,t}$, $L_{u,occ,t}$: occupation-specific skilled and unskilled labor.
- $\rho_{3,occ} \in (-\infty, 1]$: occupation-specific skill elasticities
- $\alpha_{3,occ,t}$: skill-biased technological change (share parameters)

Model: demand level 3

Level 3: Genders.

$$L_{edu,occ,t} = \left[\alpha_{4,skl,occ,t} L_{k,skl,occ,t}^{\rho_{4,occ}} + (1-\alpha_{4,skl,occ,t}) L_{f,skl,occ,t}^{\rho_{4,occ}}\right]^{1/\rho_{4,occ}}$$
$$\text{for} \quad edu = s, u,$$
$$\text{and} \quad occ = a, r, m.$$

- $L_{edu,occ,t}$: Aggregate occupation- and skill-specific labor
- $L_{k,skl,occ,t}$, $L_{f,skl,occ,t}$: occupation/skill-specific male and female labor.
- $\rho_{4,occ} \in (-\infty, 1]$: occupation-specific gender elasticities
- $\alpha_{4,skl,occ,t}$: gender-biased technological change (share parameters)

Model: Demand parameters across levels

Demand "share" parameters:

- $\boldsymbol{\alpha}_{4,t}$: Gender-biased technological change, by skill/occupation
- $\boldsymbol{\alpha}_{3,t}$: Skill-biased technological change, by occupation
- $\alpha_{2,t}$, $\alpha_{1,t}$: Task-content of occupation-biased
- Assume: $\log \alpha_{v,t} = a_{v,0} + a_{v,1} t + a_{v,2} t^2 + a_{v,3} t^3$, for $v \in \{1, 2, 3, 4\}$

Demand gender/skill/occupation labor elasticity of substitution:

- $\boldsymbol{\rho_4}$: Gender substitutability, 3 parameters
- $\boldsymbol{\rho_3}$: Skill substitutability, 3 parameters
- ρ_2, and ρ_1: Occupational substitutability

Model: Supply

Random utility framework:

- Workers choose in t among occupations home-production.
- Utility in **market occupations**: function of pecuniary (earnings) and nonpecuniary rewards + idiosyncratic random taste shock.

$$U(occ \mid gen, skl, t) = \underbrace{\psi_{gen,skl,occ}}_{\text{nonpecuniary}} + \underbrace{\psi_1 W_{gen,skl,occ,t}}_{\text{pecuniary}} + \underbrace{\epsilon_{gen,skl,occ,t}}_{\text{i.i.d. taste shock}}$$

- Utility from choosing home production is:

$$\begin{aligned} U(h \mid gen, skl, t) = & \pi_{1,gen} + \pi_{2,gen,skl} \cdot Pr(\text{child} = 1 \mid gen, skl, t) \\ & + \pi_{3,gen,skl} \cdot Pr(\text{married} = 1 \mid gen, skl, t) \\ & + \pi_{4,gen,skl} \cdot Pr(\text{appliance} = 1 \mid gen, skl, t) \\ & + \pi_{5,gen,skl} \cdot \text{WBL}_t + \epsilon_{gen,skl,h,t}, \end{aligned}$$

  - *WBL*: Women's Business and Law Index (World Bank)

The probability that a worker chooses among $O = a, r, m, h$ is:

$$Pr(d_O = 1 \mid gen, skl, t) = \frac{\exp(U(O \mid gen, skl, t))}{\sum_{occ=a,r,m,h} \exp(U(occ \mid gen, skl, t))}$$

The total labor supply of each type is, for example:

$$L^s_{f,s,a,t} = L^{\text{pop}}_{f,s,t} \times Pr(d_a = 1 \mid f, s, t)$$

- $Pr(d_a = 1 \mid f, s, t)$: Gender/skill-specific occupation rate
- $L^{\text{pop}}_{f,s,t}$ (demographics): The exogenous number of potential *female* workers with *college education* (skilled) at time t
- $L^s_{f,s,a,t}$: Gender/skill/occupation-specific quantity of labor supply

Model: Equilibrium

1. *Optimality*: wage = marginal productivities,

$$\frac{W_{male,edu,occ,t}}{W_{female,edu,occ,t}} = \frac{\partial Y_t / \partial L^d_{male,edu,occ,t}}{\partial Y_t / \partial L^d_{female,edu,occ,t}}.$$

2. *Equilibrium*: across occupation, skill and gender nests,

$$L^d_{gen,edu,occ,t} = L^s_{gen,edu,occ,t}$$

with total supply in home production as the residual.

Equilibrium Solution

For notational clarity, ignoring t and skl subscripts ...

First, demand equals supply:

$$\underbrace{L_r \cdot \left(\alpha_{k,r} + \alpha_{f,r} \left(\frac{W_{k,r}}{W_{f,r}} \frac{\alpha_{f,r}}{\alpha_{k,r}} \right)^{\frac{\rho_{4,r}}{1-\rho_{4,r}}} \right)^{\frac{-1}{\rho_{4,r}}}}_{L^d_{k,r}\text{, optimal male routine labor demand given wages}} = \underbrace{\frac{L^{pop}_k \cdot \exp\left(\widehat{U}_k\left(r \mid W_{k,r}, \boldsymbol{B}_k\right)\right)}{\sum_{o\in\{a,r,m,h\}} \exp\left(\widehat{U}_k\left(o \mid W_{k,O}, \boldsymbol{B}_k\right)\right)}}_{L^s_{k,r}\text{, optimal male routine labor supply given wages}}$$

Second, male routine wage $W_{k,r}$ as a function of female wages $\boldsymbol{W}_f$:

$$\widehat{W}_{k,r}\left(\boldsymbol{W}_f\right) = \left(\left(\frac{L_r}{L^s_{f,r}(W_{f,a}, W_{f,r}, W_{f,m}; \boldsymbol{B}_f)} \right)^{\rho_{4,r}} \frac{1}{\alpha_{k,r}} - \frac{\alpha_{f,r}}{\alpha_{k,r}} \right)^{\frac{\rho_{4,r}-1}{\rho_{4,r}}} \frac{\alpha_{k,r} W_{f,r}}{\alpha_{f,r}}$$

Third, for each $o \in \{a, r, m\}$, we have:

$$W_{f,o} = \widehat{W}_{f,o}\left(\widehat{W}_{k,a}\left(\boldsymbol{W}_f\right), \widehat{W}_{k,r}\left(\boldsymbol{W}_f\right), \widehat{W}_{k,m}\left(\boldsymbol{W}_f\right)\right)$$

Hence, given $\boldsymbol{L}$, equilibrium by skill is characterized by three equations with three unknowns (female wages), equilibrium $\boldsymbol{W}^\star\left(\boldsymbol{L}\right)$ are the roots.

Estimation I: Challenges

Estimation challenges:

1. Estimate parameters at different nest levels
2. Estimate elasticity and share parameters
3. Estimate demand parameters vs supply parameters

Given genders, skills, and occupations, the problem has:

- 12 nests $(6 + 3 + 2 + 1) = 12$
- 29 supply and 65 demand parameters
- 312 model predictions (12 W and L in each of 13 years)

Estimation II: Data variations and parameters

- We discuss identification of parameters across nests using relative wages within and across nests.
- The lowest nest directly faces observed wages and labor quantities, higher nest layers generate aggregate wages and quantities based on lower level parameters and observables.
- We discuss the data requirements for jointly identifying elasticities and demand shares - and show that identification is based on the concept of time-invariance in demand parameters after differencing
- Estimation proceeds by searching for the demand and supply side parameters that generate the best fit between equilibrium predictions and the data

Estimation III: Equilibrium estimation

At each t, we observe $\boldsymbol{W}_t$ and $\boldsymbol{L}_t$. We have

- vectors of demand parameters $\left\{\widehat{\boldsymbol{\alpha}}_{4,t}, \widehat{\boldsymbol{\alpha}}_{3,t}, \widehat{\alpha}_{2,t}, \widehat{\alpha}_{1,t}, \frac{Y_t}{Z_t}, \boldsymbol{\rho}\right\}$,
- supply parameters $\{\boldsymbol{\psi}, \boldsymbol{\pi}\}$,
- gender- and skill-specific supply-side variables $\boldsymbol{B}_t$,
- and gender/skill-specific potential workers $\boldsymbol{L}_t^{pop}$.

We solve for equilibrium vectors $\widehat{\boldsymbol{W}}_t$ and $\widehat{\boldsymbol{L}}_t$. Given normal measurement errors $\{\boldsymbol{\epsilon}_t, \boldsymbol{\eta}_t\}$, in a *gen*, *skl*, and *occ* cell, we have:

$$\log\left(L_{gen,skl,occ,t}\right) = \log\left(\widehat{L}_{gen,skl,occ,t}\right) + \eta_{gen,skl,occ,t} \ ,$$

$$\text{and} \ \log\left(W_{gen,skl,occ,t}\right) = \log\left(\widehat{W}_{gen,skl,occ,t}\right) + \epsilon_{gen,skl,occ,t} \ .$$

We estimate via GMM with score of the log likelihood.

Estimation IV: Demand or supply only estimation

Supply-side only estimation: wages are endogenous to LFP.

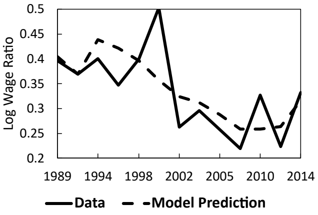
Demand-side only estimation:

- Bias from **measurement error**:
  Bias if both wage and labor are measured with error.
- Invalid **instruments**:
  When labor supply is elastic with respect to wages, supply shocks (that are uncorrelated with demand shocks) are not valid instruments because equilibrium quantities are determined by supply and demand shocks jointly.
- Unavailable **equilibrium-supply shifter**:
  A supply shock traces out the demand curve when the demand curve stays invariant. We show that our $\boldsymbol{W}$ and $\boldsymbol{L}$ changes from t to $t+1$ (every 2/4 years) can not be explained by changes in the supply curve only.

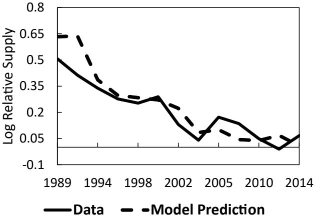
Fit I

Analytical: relative men "price" and "quantity" both down

(a) Log (M/F) Earnings Ratio

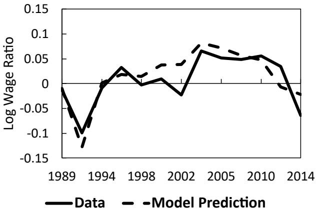


(b) Log (M/F) Relative Supply

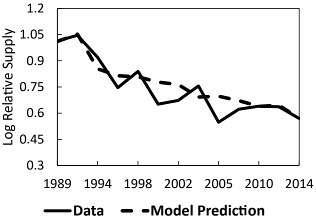


Routine: relative men "price" up/stable, "quantity" down

(c) Log (M/F) Earnings Ratio



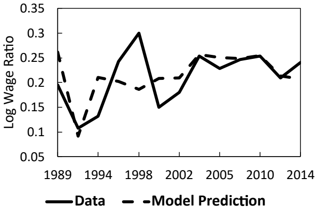
(d) Log (M/F) Relative Supply



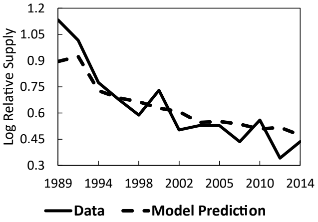
Fit II

Manual: relative men "price" up/stable, "quantity" down

(a) Log (M/F) Earnings Ratio

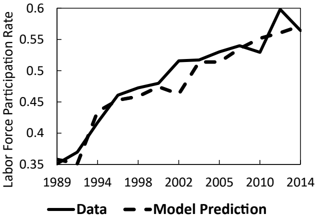


(b) Log (M/F) Relative Supply

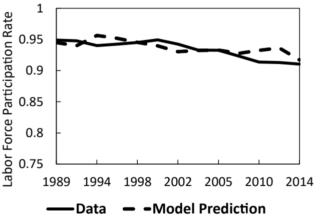


Participation Rates: aggregate gender across higher nests

(c) Female



(d) Male



Estimates I: Demand substitutability estimates

Production estimates: women substitute men more in analytical

	Elasticities of Substitution			
	Estimate	[SE]	Implied Elasticity $(1/(1-\rho))$	95% Conf. Int. $(1/(1-\rho))$
Gender				
$\rho_{4,m}$: male, female (manual)	0.084	[0.066]	1.091	[0.955, 1.273]
$\rho_{4,r}$: male, female (routine)	0.218	[0.067]	1.278	[1.093, 1.540]
$\rho_{4,a}$: male, female (analytical)	0.660	[0.078]	2.941	[2.022, 5.389]
Education				
$\rho_{3,m}$: skilled, unskilled (manual)	0.739	[0.036]	3.831	[3.010, 5.271]
$\rho_{3,r}$: skilled, unskilled (routine)	0.301	[0.110]	1.431	[1.091, 2.078]
$\rho_{3,a}$: skilled, unskilled (analytical)	0.302	[0.125]	1.433	[1.058, 2.220]
Occupation				
ρ_1: analytical, routine and manual	0.031	[0.092]	1.032	[0.869, 1.271]
ρ_2: routine, manual	-0.154	[0.159]	0.867	[0.681, 1.192]

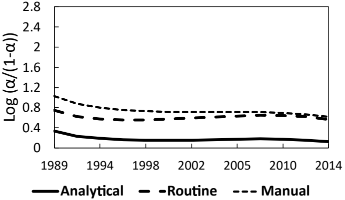
σ_4: 2.4 (Weinberg 2000), 3 (Acemoglu, Autor, and Lyle 2004), [1.8, 2.2] (Johnson and Keane 2013).

σ_3: 1.5 (Katz and Murphy 1992; Ciccone and Peri 2005; Johnson and Keane 2013), 2.1 (Manacorda, Sánchez-Paramo, and Schady 2010), 1.25 (Fernández and Messina 2018).

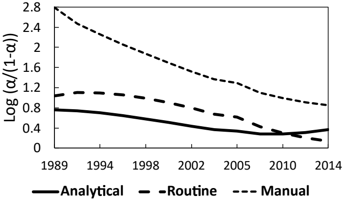
Estimates II: Demand "share" estimates

Demand Level III: relative demand for females up, more so for skilled analytical

(a) Male vs. Female (High School)

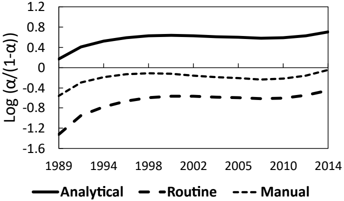


(b) Male vs. Female (College)



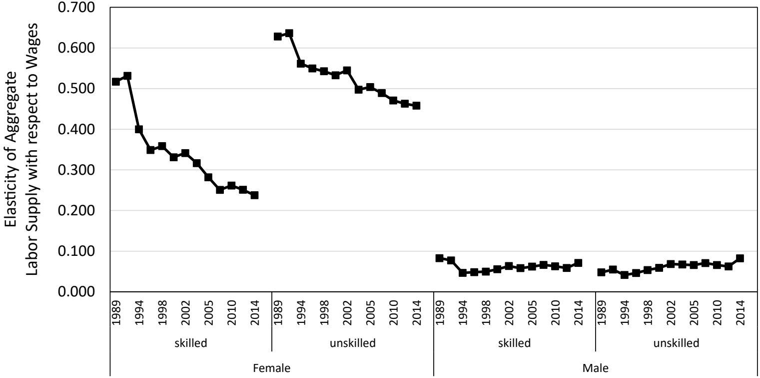
Demand Level II: relative demand for skilled up for analytical

(c) College vs. High School

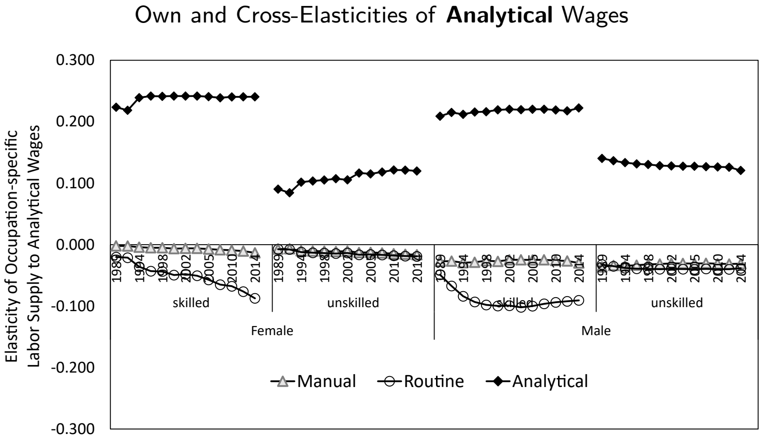


Estimates III: Supply aggregate wage elasticities

The Elasticity of Gender- and Skill-specific Aggregate Labor Supply with Respect to Wages



Estimates IV: Supply own and cross wage elasticities



Counterfactuals I: Key exogenous drivers

For counterfactual, keep each of the following exogenous variables or parameters at its 1989 value:

1. Non-wage determinants of LFP
   - Increasing appliance availability for unskilled women
   - Decreasing fertility for skilled women
2. Demographics (indirect effects on LFP)
   - Increasing skilled women among skilled potential workers
   - Increasing emigration of unskilled (male) potential workers
3. Demand parameters
   - Skill-biased technological changes favoring skilled
   - Gender-biased technological changes favoring women

Counterfactuals II: Aggregate summary

Counterfactual aggregate results summary

	Change in Gender Participation and Wage Gaps: C.2012 - C.1992						
		Non-Wage Supply		**Demographics**		**Demand**	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Model	Fertility	Appliance	Skilled Female	Emigrant	Gender α_4	Skill α_3
100 × Δ (Male - Female) LFP							
Overall **PE**	-19.9	-16.5	-7.5	-17.8	-19.6	—	—
$\left(\frac{\text{Model}-\text{Counter}}{\text{Model}} \cdot 100\%\right)$		(17%)	(62%)	(11%)	(2%)	—	—
Overall **GE**	-19.9	-17.9	-14.3	-24.2	-16.7	-16.5	-16.9
$\left(\frac{\text{Model}-\text{Counter}}{\text{Model}} \cdot 100\%\right)$		(10%)	(28%)	(-22%)	(16%)	(17%)	(15%)
100 × Δ Log (Male/Female) Wage Ratio							
Overall **PE**	-6.3	-3.3	-10.6	5.3	-6.3	—	—
$\left(\text{Counter} - \text{Model}\right)$		(3.0)	(-4.3)	(11.6)	(0.0)	—	—
Overall **GE**	-6.3	-5.6	-12.2	9.6	-12.6	17.6	-5.1
$\left(\text{Counter} - \text{Model}\right)$		(0.7)	(-5.9)	(15.9)	(-6.3)	(23.9)	(1.2)

Notes: Pink factors **decrease** male to female LFP or wage gaps.

Counterfactuals III: Fertility and appliance availability

Counterfactual fertility and appliance availability key results

	Change in Gender Participation and Wage Gaps: C.2012 - C.1992						
		Overall		**Skilled Analytical**		**Unskilled Manual**	
	(1) Model	(2) Fertility	(3) Appliance	(4) Model	(5) Fertility	(6) Model	(7) Appliance
100 × Δ (Male - Female) LFP and Occupation Rates							
Overall **PE**	-19.9	-16.5	-7.5	-3.9	-2.2	-7.6	-2.6
$\left(\frac{\text{Model}-\text{Counter}}{\text{Model}} \cdot 100\%\right)$		(17%)	(62%)		(44%)		(66%)
Overall **GE**	-19.9	-17.9	-14.3	-3.9	-2.5	-7.6	-5.4
$\left(\frac{\text{Model}-\text{Counter}}{\text{Model}} \cdot 100\%\right)$		(10%)	(28%)		(36%)		(29%)
100 × Δ Log (Male/Female) Wage Ratio							
Overall **PE**	-6.3	-3.3	-10.6		—		—
$\left(\text{Counter} - \text{Model}\right)$		(3.0)	(-4.3)		—		—
Overall **GE**	-6.3	-5.6	-12.2	-9.0	-13.2	7.1	-2.0
$\left(\text{Counter} - \text{Model}\right)$		(0.7)	(-5.9)		(-4.2)		(-9.1)

Notes: Pink factors **decrease** male to female LFP/occupation-rates or wage gaps.

Counterfactuals IV: Female skill upgrading

1. PE: Skilled female workers earn higher wages and have higher LFP, compositional reductions in the gender LFP/wage gaps
2. GE effects by sub-groups:
   - Expanding skilled female supply pushes down their wages
   - Contraction in skilled males demand pushes down their wages, due to high analytical gender-substitutability ($\rho_{4,a}$)
   - Contraction in demand for unskilled male and female, due to high manual skill-substitutability ($\rho_{3,m}$)
   - Lower unskilled wages (potentially effects on unskilled female wages is ambiguous)
   - Fall in wages has larger impact on female LFP, given larger female wage elasticities
3. GE LFP: GE effects widens gender LFP gap (reverses PE)
4. GE LFP: GE effects magnifies the PE wage gap narrowing

Counterfactuals V: Skilled analytical

Counterfactual skilled analytical summary

	Change in Gender Participation and Wage Gaps: C.2012 - C.1992						
		Non-Wage Supply		**Demographics**		**Demand**	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Model	Fertility	Appliance	Skilled Female	Emigrant	Gender α_4	Skill α_3
100 × Δ (Male - Female) Occupation Rates							
Overall **PE**	-3.9	-2.2	-3.5	0.9	-4.0	—	—
$\left(\frac{\text{Model}-\text{Counter}}{\text{Model}} \cdot 100\%\right)$		(44%)	(10%)	(123%)	(-3%)	—	—
Overall **GE**	-3.9	-2.5	-4.0	1.1	-3.7	-3.7	-2.7
$\left(\frac{\text{Model}-\text{Counter}}{\text{Model}} \cdot 100\%\right)$		(36%)	(-3%)	(128%)	(5%)	(5%)	(31%)
100 × Δ Log (Male/Female) Wage Ratio							
Overall **PE**	-9.0	—	—	—	—	—	—
(Counter − Model)		—	—	—	—	—	—
Overall **GE**	-9.0	-13.2	-10.1	-28.6	-11.5	32.3	-9.4
(Counter − Model)		(-4.2)	(-1.1)	(-19.6)	(-2.5)	(41.3)	(-0.4)

Notes: Pink factors **decrease** male to female occupation-rates or wage gaps.

Counterfactuals VI: Unskilled manual

Counterfactual unskilled manual summary

	Change in Gender Participation and Wage Gaps: C.2012 - C.1992						
		Non-Wage Supply		**Demographics**		**Demand**	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Model	Fertility	Appliance	Skilled Female	Emigrant	Gender α_4	Skill α_3
100 × Δ (Male - Female) Occupation Rates							
Overall **PE**	-7.6	-7.1	-2.6	-9.5	-7.4	—	—
$\left(\frac{\text{Model} - \textit{Counter}}{\text{Model}} \cdot 100\%\right)$		(7%)	(66%)	(-25%)	(3%)	—	—
Overall **GE**	-7.6	-7.4	-5.4	-11.3	-6.6	-5.8	-7.3
$\left(\frac{\text{Model} - \textit{Counter}}{\text{Model}} \cdot 100\%\right)$		(3%)	(29%)	(-49%)	(13%)	(24%)	(4%)
100 × Δ Log (Male/Female) Wage Ratio							
Overall **PE**	7.1	—	—	—	—	—	—
$\left(\text{Counter} - \text{Model}\right)$		—	—	—	—	—	—
Overall **GE**	7.1	8.2	-2.0	25.2	-1.8	22.9	3.9
$\left(\text{Counter} - \text{Model}\right)$		(1.1)	(-9.1)	(18.1)	(-8.9)	(15.8)	(-3.2)

Notes: Pink factors **decrease** male to female occupation-rates or wage gaps.

Robustness

- Including earnings from part-time workers. ▸ Show
- Using hours worked as measure of labor supply. ▸ Show
- Changing order of nests in the production technology. ▸ Show
- Changing order of occupations in level 1 of production technology. ▸ Show

Conclusions

- Structurally estimate an equilibrium model of supply and demand for labor using Mexican data over the last two decades.
- Model incorporates ideas of the task based approach. Allows us to think about substitutability of labor by gender and education across occupations.
  - Elasticity of substitution between male and female labor is heterogeneous across the earnings distribution.
  - Higher substitutability in abstract task-intensive occupations.

Robustness I

Table: Parameter Estimates: Production Technology. Alternative Supply/Earnings Measures

	Full-Time Workers			Part-Time Workers			Hours Worked		
	Estimate	SE	Elasticity	Estimate	SE	Elasticity	Estimate	SE	Elasticity
Gender									
$\rho_{4,m}$: female, male (manual)	0.175	[0.181]	1.212	-0.258	[0.152]	0.795	0.161	[0.138]	1.192
$\rho_{4,r}$: female, male (routine)	0.179	[0.129]	1.219	-0.030	[0.110]	0.971	0.355	[0.146]	1.551
$\rho_{4,a}$: female, male (abstract)	0.622	[0.099]	2.646	0.607	[0.121]	2.543	0.666	[0.108]	2.990
Education									
$\rho_{3,m}$: college, secondary (manual)	0.722	[0.067]	3.594	0.771	[0.083]	4.371	0.803	[0.120]	5.081
$\rho_{3,r}$: college, secondary (routine)	0.355	[0.041]	1.549	0.364	[0.073]	1.572	0.342	[0.122]	1.519
$\rho_{3,a}$: college, secondary (abstract)	0.276	[0.121]	1.382	0.151	[0.197]	1.177	0.173	[0.211]	1.209
Occupation									
ρ_1: abstract, routine and manual	0.031	[0.094]	1.032	0.688	[0.167]	3.206	0.621	[0.186]	2.639
ρ_2: routine, manual	-0.141	[0.183]	0.877	-0.519	[0.146]	0.658	-0.246	[0.192]	0.803

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Robustness II

Table: Parameter Estimates: Production Technology. Alternative Model Specifications

	Baseline			Nests Order Swap			Routine			Manual		
	Estimate	SE	Elasticity	Estimate	SE	Elasticity	Estimate	SE	Elasticity	Estimate	SE	Elasticity
Gender												
$\rho_{4,m}$: female, male (manual)	0.175	[0.181]	1.212	-0.246	[0.098]	0.802	-0.427	[0.177]	0.701	-0.029	[0.104]	0.972
$\rho_{4,r}$: female, male (routine)	0.179	[0.129]	1.219	-0.278	[0.102]	0.782	-0.095	[0.153]	0.913	0.007	[0.023]	1.007
$\rho_{4,a}$: female, male (abstract)	0.622	[0.099]	2.646	0.466	[0.115]	1.872	0.529	[0.099]	2.121	0.551	[0.099]	2.225
Education												
$\rho_{3,m}$: college, secondary (manual)	0.722	[0.067]	3.594	0.564	[0.045]	2.292	0.454	[0.104]	1.831	0.581	[0.058]	2.385
$\rho_{3,r}$: college, secondary (routine)	0.355	[0.041]	1.549	0.382	[0.054]	1.618	0.380	[0.051]	1.614	0.189	[0.047]	1.233
$\rho_{3,a}$: college, secondary (abstract)	0.276	[0.121]	1.382	0.012	[0.040]	1.012	0.008	[0.048]	1.008	0.446	[0.150]	1.805
Occupation												
ρ_1: abstract, routine and manual	0.031	[0.094]	1.032	0.441	[0.109]	1.788						
ρ_2: routine, manual	-0.141	[0.183]	0.877	-1.816	[0.135]	0.355						
ρ_1: routine, abstract and manual							-0.784	[0.231]	0.560			
ρ_2: abstract, manual							0.332	[0.171]	1.496			
ρ_1: manual, abstract and routine										0.411	[0.134]	1.697
ρ_2: abstract, routine										-0.714	[0.099]	0.583

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Let $q_\tau(F_W)$ be τth quantile of the distribution of wages, expressed in terms of the cumulative distribution $F_W(w)$. Define the following mixture distribution:

$$G_{W,\epsilon} = (1-\epsilon)F_W + \epsilon H_W \quad \textit{for} \quad 0 \leq \epsilon \leq 1, \tag{10.1}$$

where H_W is some perturbation distribution that only puts mass at the value w. By definition, the influence function corresponds to:

$$IF(w; q_\tau, F_W) = lim_{\epsilon \to 0} \; \frac{q_\tau(G_{W,\epsilon}) - q_\tau(F_W)}{\epsilon}, \tag{10.2}$$

where the expression is analogous to the directional derivative of q_τ in the direction of H_W. By definition, the expectation is equal to zero.

$$\int_{-\infty}^{+\infty} IF(w; q_\tau, F_W) dF(w) = 0, \tag{10.3}$$

Firpo, Fortin, and Lemieux (2009) propose a simple modification in which the quantile is added back to the influence function, resulting in what the authors call the Recentered Influence Function (RIF).

$$RIF(w; q_\tau, F_W) = q_\tau + IF(w; q_\tau, F_W) \tag{10.4}$$

We can model the conditional expectation of the RIF as a linear function of the explanatory variables.

$$E[RIF(w; q_\tau, F_W|X)] = X'\beta. \tag{10.5}$$

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Task-Based Approach I

Complementary or substitutability between factors of production is determined by the type of tasks in which they are employed (Autor, Levy, and Murnane 2003; Autor, Katz, and Kearney 2006; Goos and Manning 2007; Dorn 2009; Rendall 2013; Autor and Dorn 2013; Adda, Dustmann, and Stevens 2017).

- Cognitive, manual, physical, socio-emotional, and interpersonal skills are applied to specific tasks in different intensities.
- Relative importance of any subset of skills is mostly determined by the nature of the activities being done.

Typology of Occupations:

1. **Abstract task-intensive:** Skills like quantitative reasoning, direction, control, and planning of activities. High-paying occupations (e.g. professional and managerial).
2. **Routine task-intensive:** Require aptitudes like adaptability to repetitive work, and a mixture of physical and analytical demands. Middle of the pay distribution (e.g. clerical and repetitive production).
3. **Manual task-intensive:** Skills like strength and hand, eye, and foot coordination. Low-paying occupations (e.g. agriculture and services).

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Simplified error structure (Johnson and Keane 2013).

$$e_i = q_i - p_i(\Theta^*), \tag{10.6}$$

- Θ: 81 $\times$ 1 vector of parameters.
- $p(\Theta)$: 364 $\times$ 1 vector formed by the wage and supply predictions of the model as function of the parameters.
- q: be the observed vector of wages and supplies derived directly from the ENIGH.

Errors e_i are assumed to be *iid* and follow a normal distribution.

$$f(e_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} exp(\frac{e_i^2}{2\sigma_i^2})$$

We use the score function to generate the moments.

The log-likelihood function takes the form:

$$\log \mathcal{L}(\Theta) = \sum_i \log f(e_i) = \sum_i \log f(q_i - p_i(\Theta)), \tag{10.7}$$

and the respective score function, $s(\Theta)$, is:

$$s(\Theta) = \frac{\partial \log \mathcal{L}(\Theta)}{\partial \Theta} = \sum_i \frac{\partial \log f(q_i - p_i(\Theta))}{\partial \Theta} = \sum_i \frac{-1}{\sigma_i^2} \frac{\partial p_i(\Theta)}{\partial \Theta}(q_i - p_i(\Theta)), \tag{10.8}$$

which we can write more compactly in vector form as

$$s(\Theta) = W'(\Theta)(q - p(\Theta)). \tag{10.9}$$

$$s(\hat{\Theta}_{ml}) = W'(\hat{\Theta}_{ml})(q - p(\hat{\Theta}_{ml})) = 0. \tag{10.10}$$

$m = q - p(\Theta)$ as a vector of population moments such that $E(q - p(\Theta)) = 0$.

$$g(\hat{\Theta}_{gmm}) = W'(q - p(\hat{\Theta}_{gmm})) = 0, \tag{10.11}$$

$$\hat{\Theta}_{gmm} = \text{argmin}(q - p(\Theta))' W(\Theta) W'(\Theta)(q - p(\Theta)). \tag{10.12}$$

Standard errors: let Γ be the matrix of partial derivatives of the sample moments $\bar{m}(\hat{\Theta}_{gmm})$ with respect to the parameters. The ith row correpsonds to:

$$\Gamma_i(\hat{\Theta}_{gmm}) = \frac{\partial \bar{m}_i(\hat{\Theta}_{gmm})}{\partial \hat{\Theta}_{gmm}}, \tag{10.13}$$

so the variance-covariance matrix can be calculated using:

$$\hat{Var}(\hat{\Theta}_{GMM}) = \{\Gamma(\hat{\Theta}_{gmm})' \{\hat{Var}[\bar{m}(\Theta_{gmm})]\}^{-1} \Gamma(\hat{\Theta}_{gmm})\}^{-1} \tag{10.14}$$

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